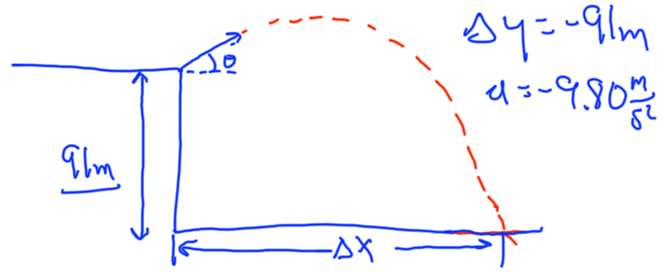
Reminders 07-23-09:

- *R*ead Chapter 6 and 7; (***make sure you thoroughly read through the chapters we cover)***
- Volunteers for Mr. Randall 2PM S-105
- 5th Webassign due Thursday 11:59PM
- Hand in 4th Assignment Problems Today
- ***Exam 3 Chapters 6-8 Next Thursday***

Objectives:

- Uniform Circular Motion
- Centripetal Acceleration
- Centripetal Force
- Gravitation

- A projectile is launched from a 91 m high cliff at an angle 37° above the horizontal. The initial velocity is 35m/s.
  - Draw a picture that is representative of this problem.
  - How do you calculate the time of flight?
  - How do you calculate the range of the flight?
  - How would you calculate the velocity of the projectile when it hits the ground below?



$\Delta y = -91\,m$

$a = -9.80\,\frac{m}{s^2}$

$V_{ix} = 35\,m/s \cos 37°$

$V_{iy} = 35\,m/s \sin 37°$

$\Delta y = V_i \sin\theta t + \frac{1}{2} a t^2$

$\Delta y = V_i \sin\theta t - \frac{1}{2}(9.80\,m/s^2) t^2$

$-91\,m = (35\,\frac{m}{s}) \sin 37° t - (4.9\,\frac{m}{s^2}) t^2$

$-91\,m = (21\,\frac{m}{s}) t - (4.9\,\frac{m}{s^2}) t^2$

$(4.9\,\frac{m}{s^2}) t^2 - (21\,\frac{m}{s}) t - 91\,m = 0$

$$t = \frac{21\,\frac{m}{s} \pm \sqrt{(21\,\frac{m}{s})^2 - 4(4.9\,\frac{m}{s^2})(-91\,m)}}{9.8\,m/s^2}$$

$t = 7.0\,s$

$\Delta X = V_{ix} t = V_i \cos 37 t$

$= (35\,m/s) \cos 37\,(7.0\,s)$

$= 196\,m$

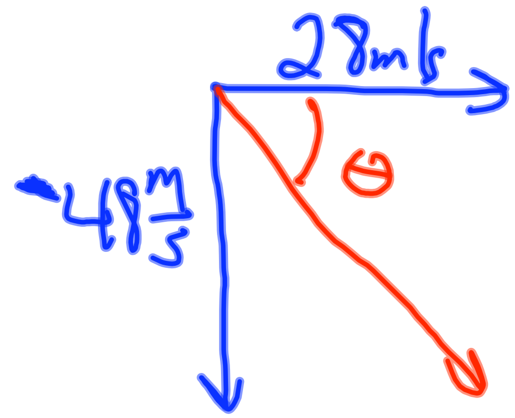
$V_{xf} = (35\,m/s) \cos 37° = 28\,m/s$

$V_{xf} = 28\,m/s = V_{xi}$

$V_{yf} = ?$ $\qquad V_{yf} = V_{yi} + at$

$V_{yf} = (21\,m/s) - (9.80\,\frac{m}{s^2})(7.0\,s)$

$= -48\,m/s$

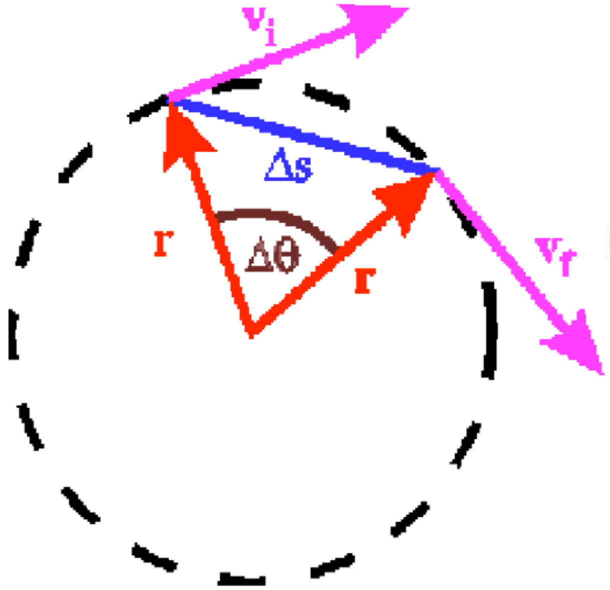


$V = \sqrt{(28\,\frac{m}{s})^2 + (48\,\frac{m}{s})^2}$

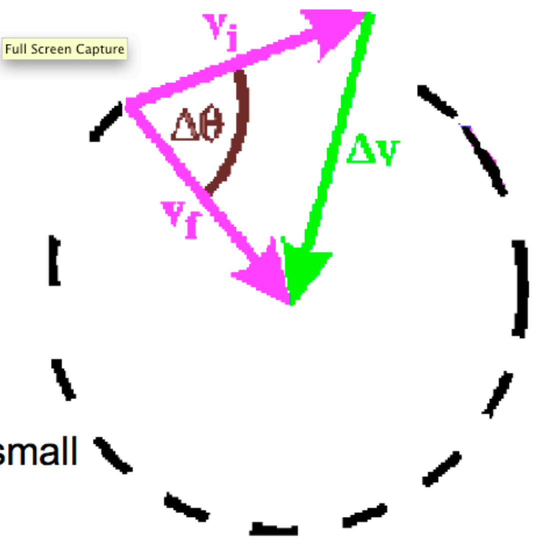
$= 56\,m/s$

$\theta = \tan^{-1}\left(\frac{-48\,m/s}{28\,m/s}\right) = -60^\circ$

60° below x-axis



Note: Δ**v** points toward the center of the circular path.



Note $\Delta v/v=\Delta s/r=v\Delta t/r$ if Δt is very small
Rearranging, this yields

$$a_c = \frac{v^2}{r}$$

Why are the angles equal?

$$\frac{V\Delta t}{R} = \frac{\Delta V}{V}$$

$$\frac{V^2}{R} = \frac{\Delta V}{\Delta t} \quad \text{by def.}$$

$$a = \Delta V/\Delta t$$

$$a = V^2/R$$

for uniform circular motion

$$V_{avg} = \frac{\Delta x}{\Delta t} = |\vec{V}|$$

$\Delta x = 2\pi R$

Δt = time it takes to complete circle. This is called the period, T.

T is # sec per revolution

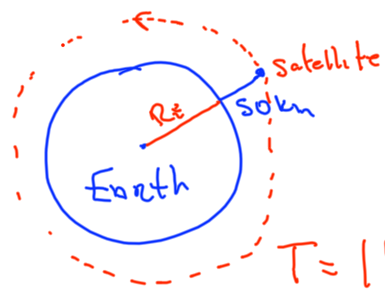
f is # revolutions per second, this is called frequency

$$f = \frac{1}{T}$$

$$V = \frac{2\pi R}{T} = 2\pi R f$$

Examples

- A satellite is in a circular orbit 5.0x10¹km above the Earth's surface. It makes one revolution in 1hr and 35 minutes. What is the centripetal acceleration of the satellite? What if the orbit is geo-synchronous?



$$r = R_E + 50\text{km}$$
$$= 6370\text{km} + 50\text{km}$$
$$= 6420\text{km}$$
$$= 6.420 \times 10^6 \text{ m}$$

$$T = 1\text{hr } 35 \text{ min}$$
$$= 95\text{min}.$$
$$= 95\text{min} \cdot \frac{60\text{s}}{\text{min}} = 5700\text{s}$$

$$a = \frac{v^2}{r} = \left(\frac{2\pi r}{T}\right)^2 \frac{1}{r} = \frac{4\pi^2 r^2}{T^2} \cdot \frac{1}{r}$$
$$= \frac{4\pi^2 r}{T^2}$$

$$a = \frac{4\pi^2 (6.420 \times 10^6 \text{m})}{(5700)^2} = 7.8 \text{ m/s}^2$$

---

geosychronous orbit

$$T = 1 \text{ day}$$
$$= (24 \cdot 60 \cdot 60) = 86{,}400$$

$$a = \frac{4\pi^2 (6.420 \times 10^6 \text{m})}{(86{,}400\text{s})^2} = .034 \text{ m/s}^2$$

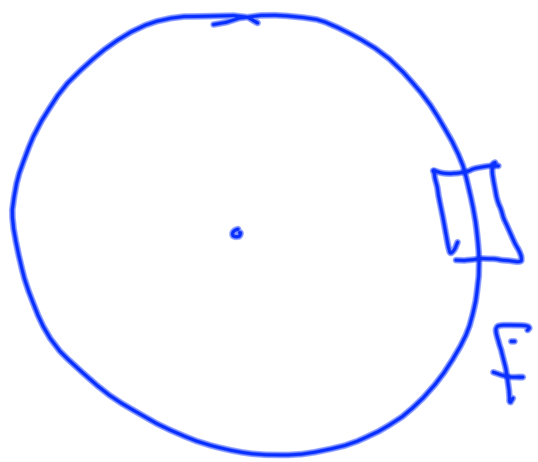
- A 0.12 kg object attached to a string is whirled in a horizontal circle whose radius is 0.75m. The velocity of the object is 3.0m/s. What is the centripetal acceleration and the force acting on the object?

$$a_c = \frac{v^2}{r} = \frac{(3.0\,m/s)^2}{.75\,m} = 12\,m/s^2$$

net force

$$F_{net} = m a_c$$
$$= (.12\,kg)(12\,m/s^2)$$
$$= 1.4\,N$$

- An automobile is rounding a turn of constant radius of curvature. A passenger notices that the arm rest is pushing toward the center of the turn with a constant force. The passenger has a mass of 78 kg. The force of the armrest on him is 150 N. The forward speed of the automobile is 21 m/s.
  - What is the acceleration of the car?
  - What is the radius of the turn?



150N is centripetal force

$F = ma$

$$a_c = \frac{F}{m} = \frac{150\,N}{78\,kg} = 1.9\,m/s^2$$

$$a_c = \frac{v^2}{r} \qquad r = \frac{v^2}{a_c} = \frac{(21\,m/s)^2}{1.9\,m/s^2}$$

$$\underline{r = 230\,m}$$

Examples

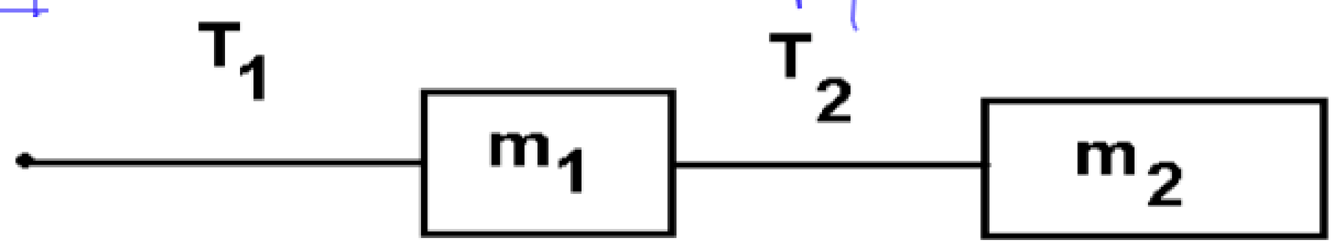
Consider the following scenerio.

The masses are moving in a circular path.

$T_1 \quad m_1 \quad T_2$

$$T_1 - T_2 = \frac{m_1 V_1^2}{r_1}$$

V

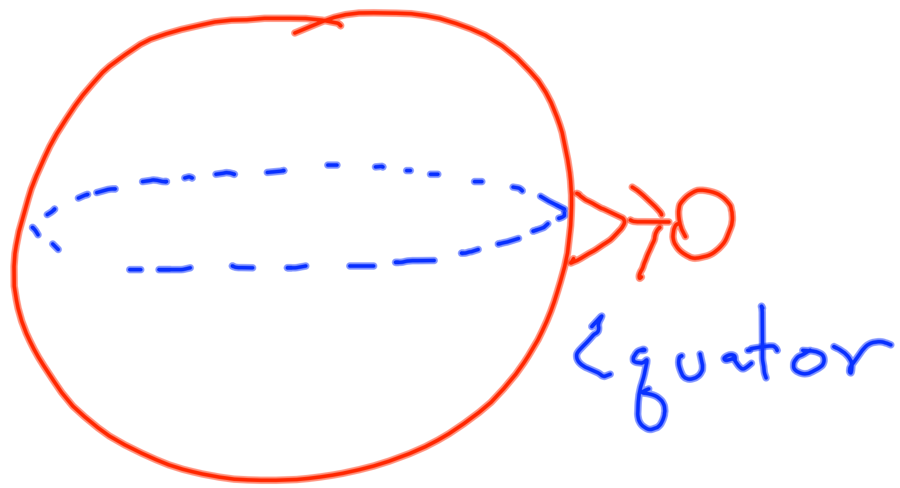


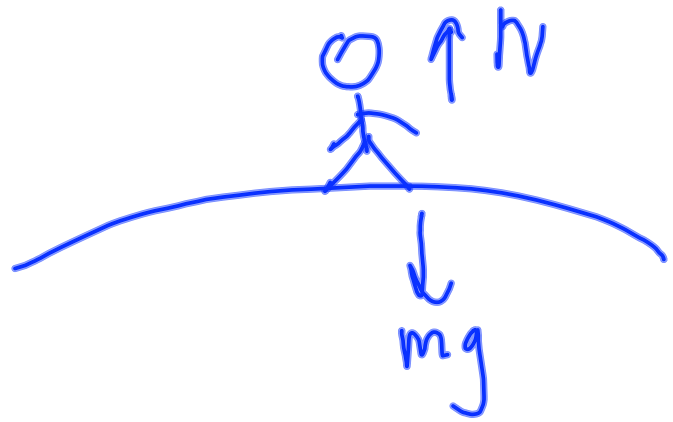
Which string is most likely to break first?

$$T_1 = T_2 + \frac{m_1 V_1^2}{r_1}$$

$T_2 \quad m_2$

$$T_2 = \frac{m_2 V_2^2}{r_2}$$





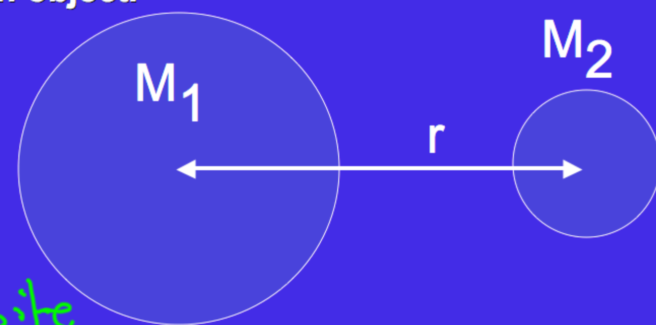
$$N - mg = -\frac{mv^2}{r}$$

$$\boxed{N = mg - \frac{mv^2}{r}}$$

Review

- Consider the system below
  - Draw arrows indicating the direction and the magnitude of the gravitational force acting on each object.

forces equal but opposite



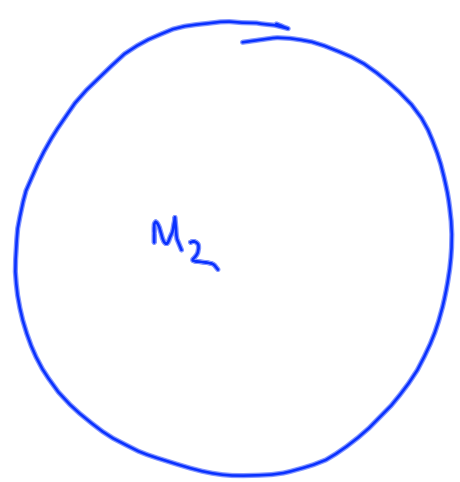
Newton's Law of gravity

$$F = \frac{G M_1 M_2}{r^2}$$

r is distance between objects

$$G = 6.67 \times 10^{-11} \frac{N\,m^2}{Kg^2}$$

M_1



Forces are equal but opposite

- A 900 N person goes to a planet whose mass is twice that of the Earth, and whose radius is three times that of the Earth. What will the person's weight be on this planet? What about the person's mass?

$$F_e = \frac{G M_e m_p}{R_E^2}$$

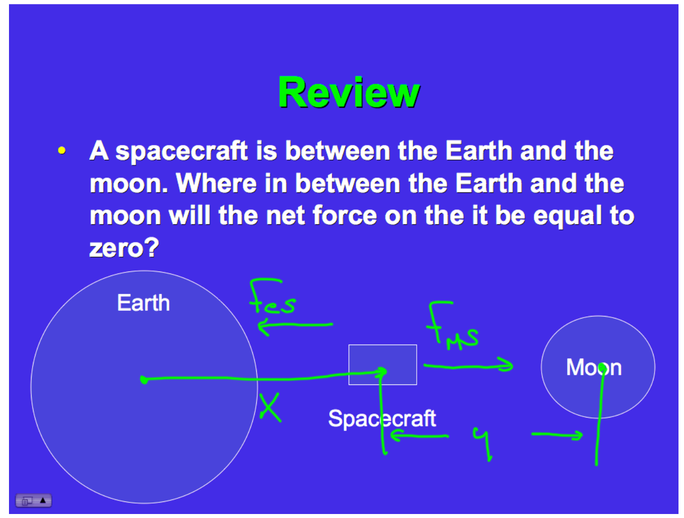
$$F_p = \frac{G (2M_e) m_p}{(3R_E)^2}$$

$$\frac{F_p}{F_e} = \frac{\dfrac{\cancel{G}(2\cancel{M_E})\cancel{m_p}}{(3R_E)^2}}{\dfrac{\cancel{G}(\cancel{M_E})\cancel{m_p}}{R_E^2}}$$

$$\frac{F_p}{F_e} = \frac{\dfrac{2}{9\cancel{R_E^2}}}{\dfrac{1}{\cancel{R_E^2}}} = \frac{2}{9}$$

$$\boxed{F_p = \frac{2}{9} F_E}$$

$$g_{planet} = \frac{2}{9} g_{earth}$$



$$F_{es} = F_{ms}$$

$$\frac{G M_E m_s}{x^2} = \frac{G M_m m_s}{y^2}$$

distance from Earth to moon d.

$$x + y = d$$

$$y = d - x$$

$$\frac{\cancel{G} M_E \cancel{m_s}}{x^2} = \frac{\cancel{G} M_m \cancel{m_s}}{(d-x)^2}$$

$$\frac{M_E}{x^2} = \frac{M_m}{(d-x)^2}$$

$$M_E (d-x)^2 = M_m x^2$$

$$\sqrt{M_e}\,(d-x) = \sqrt{M_m}\, x$$

$$d\sqrt{M_E} = \left(\sqrt{M_m}\right)x + \left(\sqrt{M_e}\right)x$$

$$= x\left[\sqrt{M_m} + \sqrt{M_e}\right]$$

$$x = \frac{d\sqrt{M_E}}{\sqrt{M_m} + \sqrt{M_E}}$$

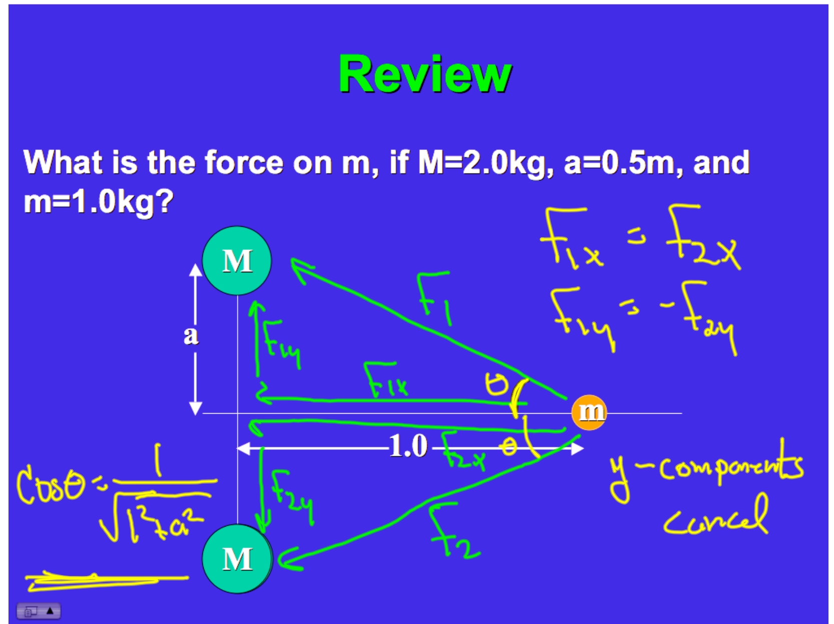
$$X = \frac{d\sqrt{M_E}}{\sqrt{M_m} + \sqrt{M_E}}$$

$$M_E = 5.98 \times 10^{24}\ \text{kg}$$

$$M_m = 7.22 \times 10^{22}\ \text{kg}$$

$$d = 3.84 \times 10^{8}\ \text{m}$$

$$X = 3.46 \times 10^{8}\ \text{m}$$



I just need to add the x-components to solve the problem

$$F_T = F_{1x} + F_{2x} = 2F_{1x}$$

$$= \frac{2GMm}{(\sqrt{1^2+a^2})^2}\cos\theta$$

$$= \frac{2GMm}{1+a^2}\frac{1}{\sqrt{1^2+a^2}}$$

$$= \left(\frac{2GMm}{1+a^2}\right)\left(\frac{1}{\sqrt{1+a^2}}\right)$$

$$= \frac{2GMm}{(1+a^2)^{3/2}}$$