Significant Figures

Introduction

The treatment of uncertainty in measurement and significant figures by most science texts is usually a practical one with performance as its sole objective. The result is usually a list of rules and examples with very little explanation. In the absence of an adequate understanding though, students often get into trouble. Rules alone can be confusing and easily forgotten. The following is intended to be a thorough presentation of this important topic.

Scientific Notation

Any number can be expressed as a number greater than or equal to 1 and less than 10 multiplied by a power of ten:

a) $100 = 1 \times 100 = 1 \times 10 \times 10 = 1 \times 10^{2}$

b) $10{,}000 = 1 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^{4}$

c) $500 = 5 \times 10 \times 10 = 5 \times 10^{2}$

d) $5976 = 5.976 \times 1000 = 5.976 \times 10^{3}$

e) $0.01 = 1 \times 10^{-2}$

f) $0.005 = 5 \times 10^{-3}$

g) $0.00087 = 8.7 \times 10^{-4}$

h) $0.0000000187 = 1.87 \times 10^{-8}$

Notice that in all of the above examples the number which is multiplied by a power of ten is a number [n] such that: $1 \leq n \leq 10$. These numbers are said to be written in scientific notation form and provide a useful way to express significant figures.

Calculations Involving Numbers in Scientific Notation Form

a) <u>Multiplication</u>

When multiplying two identical numbers rose to a power, the following general rule is applied:

$$a^{m} \times a^{n} = a^{m+n}$$

$$\text{Example: } 10^{6} \times 10^{3} = 10^{6+3} = 10^{9}$$

The same rule is followed when multiplying two numbers in scientific notation form because the numbers that are raised to a power are identical (namely 10):

$$(2 \times 10^{3})(2 \times 10^{3}) = 2 \times 2 \times 10^{3} \times 10^{3} = 4 \times 10^{6}.$$

b) Division

When dividing two identical numbers rose to a power, the following general rule is followed:

$$\frac{a^m}{a^n} = a^{m-n}$$

The above rule also applies to numbers expressed in scientific notation form as follows:

$$\frac{6 \times 10^9}{3 \times 10^3} = \frac{6}{3} \times \frac{10^9}{10^3} = 2 \times \frac{10^9}{10^3} = 2 \times 10^6$$

c) Addition and Subtraction

When adding and subtracting numbers written in scientific notation form remember the following:

1) Write all numbers involved to the same power of ten.
2) Line up the decimal points.
3) Carry out the addition or subtraction (the answer will have the same power of ten).
4) Write the answer in scientific notation form.

Example: $(7.950 \text{ x } 10^4) + (3.81 \text{ x } 10^3)$; $(3.81 \text{ x } 10^3 = .381 \text{ x } 10^4)$

$$\begin{array}{r} 7.950 \times 10^4 \\ +.381 \times 10^4 \\ \hline 8.331 \times 10^4 \end{array}$$

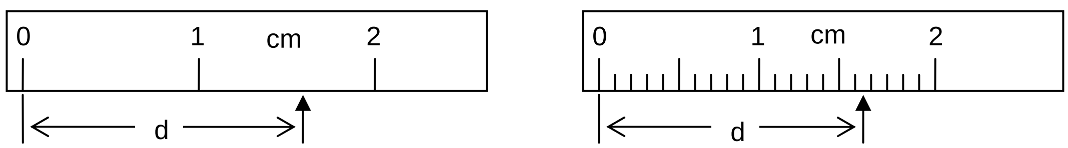
Significant Figures

There are three types of numbers generally encountered in scientific work: counting numbers, defined numbers, and measuring numbers. Counting numbers are, by their very nature, exact. Three people, four cars, etc. Defined numbers are those which are the result of arbitrary definition. Example: 12 inches = 1 foot. These numbers are also exact. Measuring numbers (measurements) are not exact because they are the result of using instruments, and no instrument can be read exactly. The way in which these inexact numbers are handled in scientific work is very important as far as the outcome of experiments is concerned. The inquiry method (scientific method) involves inductive reasoning (drawing general conclusions from specific facts). Conclusions which result from this method are only reliable within the limits set by the conditions under which the facts were obtained.

One possible condition is the nature of any instrument used. All measurements have uncertainty associated with them, so instruments can vary in the amount of uncertainty they generate. Conclusions drawn from measurements can then have a varying degree of reliability. When considering conclusions, it is desirable to know how reliable they are. One should consider all relevant conditions. But how does one deal with the conditions imposed by any instrument involved, i.e., how does one know how uncertain a measure is? There are several important ways. One is by the use of significant figures. Before we can understand them, more information about uncertainty is needed.

There are two ways in which a measurement can be uncertain. The first way is how the measurement compares to its "true value." This comparison is called the accuracy of the measurement. The idea of a "true value" is somewhat complicated by the fact that they can only be obtained by measurement (which has uncertainty), either directly or indirectly. Usually the value obtained by the "best" experiment (or instrument) is considered the "true value". The accuracy of a measurement is usually assumed to be good, for Instruments should be calibrated before use. The second way a measurement can be uncertain has to do with the instrument itself, and therefore is the one we are concerned with when considering a conclusion's reliability. What is this second kind of uncertainty? Consider the following two rulers A and B:

Ruler A | Ruler B



d = 1.6 cm | d = 1.64 cm

Clearly, ruler B's measurement of d is less uncertain than ruler A's. This uncertainty is generated whenever we read the scale of an instrument. A measure of this type of uncertainty is known as the precision of the instrument. This precision can be obtained by either looking at the instruments' calibration, or by looking at the number of reported digits. Because the instruments involved are generally not available when reading someone else's data, scientists have recognized the usefulness of the later method. Some logically based rules have to be followed by everyone.

RULE 1: When reporting measurements, the number of digits will be the number of known digits plus one doubtful one.

For example, consider ruler B again. Notice that one can be sure of the 1.6 cm, but not of the .04 cm that was estimated. Someone else may report 1.65 cm under the same conditions. The 4 or 5 is said to be the doubtful digit. You know it has some degree of uncertainty. If all measurements are reported in this way, then one can always tell where the uncertainty is and to what degree. These reported digits are called significant figures. When reading a measurement from an instrument's scale then, always estimate and record a doubtful digit from between the smallest calibration lines on the instrument. If you are uncertain about this last statement, seek clarification before continuing.

The definition of precision can be stated in another way:

"The degree to *which* a *series of measurements*, *made by* a *particular instrument,* of a *particular quantity, agree with one another.* "

For example, measurements made with rulers A and B by a group of experimenters of a particular quantity may look like the following:

Ruler A	Ruler B
1.7 cm.	1.65 cm.
1.6	1.64
1.8	1.66
1.5	1.67

Measurements made by ruler A clearly do not agree with one another as well as those made by ruler B. B has greater precision. Significant figures can give a measure of accuracy also, but only if the instrument is accurate. Consider the following true statements:

1. An instrument can be very precise, but very inaccurate.
2. An instrument which is very accurate must also be very precise.

We can see that precision and accuracy do not always go hand in hand.

The use of significant figures to reflect an instrument's precision (and hopefully its accuracy) is fairly simple until the experimenter begins to manipulate measurements. For example, consider changing the units of a measurement. Suppose 1.65 cm was changed to meters:

1.65 cm x $\frac{1m}{100cm}$ = 0.0165 m; or to microns: 1.65 cm x 10^4 microns/cm = 16,500 microns

It appears that just by changing units, we can change the precision of the instrument. This confusion can be avoided by using the following rule:

RULE 2: Use scientific notation to indicate the number of significant figures.

The above two measurements would be more conveniently expressed as: 1.65 x 10^{-2} m and 1.65 x 10^4 microns. Now the measurements show 3 significant figures instead of 4 and 5. Another popular convention gives rules regarding zeroes and significant figures. They are:

1. Zeroes to the left of a number never count [0.00179]. 2. Zeroes between two numbers always count [107].
3. Zeroes to the right of the number and to the right of the decimal point always count [16.50 and 0.1650].
4. Zeroes to the right of the number and to the left of the decimal point may or may not count, therefore this must be specified by the user [16,500].

Use scientific notation and you won't get into trouble, but also know the above rules in case someone else practices them. Suppose an instrument is actually capable of measuring a quantity like .0019 m directly. Shouldn't the precision of this instrument be indicated by 4 significant figures? If this were allowed in such cases, someone looking at the measurements would not know if they were the result of a unit change or the instrument itself. Since it does not affect comparisons between instruments, the zeroes are not counted as significant to avoid confusion.

Another point of confusion arises when one realizes that the number of significant figures that an instrument is capable of may change in going from one portion of an instrument's scale to another. For example, from 1.00 - 9.99 the instrument is capable of giving three significant figures, but from 10.00 to 99.99 it is capable of four. Likewise, below 1 it is capable of only two. Do different portions have different degrees of precision? YES! But the precision of these different portions is relative to the quantity being measured. Therefore, when measurements made by different instruments are compared for precision, they are for the same quantity.

Significant Figures and Derived Quantities

It is always desirable to carry the information given by significant figures over into numbers which are calculated from measurements. Such numbers are called derived quantities. Derived quantities are expressed in terms of the correct number of significant figures as determined by the measurements used to derive the quantity. The following example shows how measurements will determine the number of significant figures in a derived quantity. Data for the length, width, and depth of a laboratory sink are given below.

trial	length	width	depth	volume
1	43.3 cm	14.5 cm	18.5 cm	11,615.225 cm^3
2	43.6	14.6	18.5	11,712.704
3	43.5	14.3	18.3	11,383.515
4	43.2	14.8	18.7	11,956.032

Each of these data was given to 3 significant figures. They varied in the third figure. Observe that the calculated volumes agree to the first two figures, and vary in the third as well. Since a doubtful number times any number can only result in a doubtful answer, it is easy to see why these final answers can only be quoted to 3 significant figures. Trial 1: 11.6×10^3 cm^3, etc. Rules 3 and 4 are used to reflect the uncertainty in measurements corresponding to derived quantities.

RULE 3: The number of significant figures in a derived quantity that is the result of multiplication or division is equal to the number of significant figures in the ~ precise measurement used to derive the quantity.

RULE 4: The number of significant figures in a derived quantity that is the result of addition or subtraction is obtained by lining up the decimal points, carrying out the operation, and keeping only one doubtful digit in the resulting quantity by rounding off the next doubtful digit.

If the measurements are expressed in scientific notation, you must convert them to the same power of ten first. Some examples:

a) 3.1 cm x 2 cm = 6 cm2   b) (6.63 in) (4.0 x 102 in) = 2.6 x 103 in2

c) $\frac{8.2g}{2.00cm^3} = 4.1g/cm^3$ d) 6.625 cm + 2.4 cm = 9.025 cm = 9.0 cm   e) 7.43×10^2 cm − 0.110×10^2 cm = 7.32×10^2 cm

<u>ROUNDING OFF</u>

When dropping digits to properly express the precision of derived quantities, the digit just to the right of the doubtful digit is considered important statistically. Generally, the derived quantity is rounded off according to the following rule:

RULE 5: When rounding off the doubtful digit, look at the digit just to the right of it. If this digit is greater than 5, round the doubtful digit up to the next number. If it is less than 5, round down. If it is 5, round the doubtful digit off to an even number.

Some examples:
a) 6.2<u>5</u>6 cm = 6.26 cm   b) 3.7<u>8</u>45 grams = 3.78 grams
c) 9.<u>5</u>5 kg = 9.6 kg   d) 9.<u>4</u>5 kg = 9.4 kg

A **<u>Summary</u>** of the rules for significant figures:

RULE 1: When reporting measurements, the number of digits will be the number of known digits plus one doubtful one.
RULE 2: Use scientific notation to indicate the number of significant figures.
RULE 3: The number of significant figures in a derived quantity that is the result of multiplication or division is equal to the number of significant figures in the ~ precise measurement used to derive the quantity.
RULE 4: The number of significant figures in a derived quantity that is the result of addition or subtraction is obtained by lining up the decimal points, carrying out the operation, and keeping only one doubtful digit in the resulting quantity by correctly rounding off.
RULE 5: When rounding off the doubtful digit, look at the digit just to the right of it. If this digit is greater than 5, round the doubtful digit up to the next number. If it is less than 5, round down. If the it is 5, round the doubtful digit off to an even number.

RULES for ZEROES:

1. Zeroes to the left of a number never count [0.00179].
2. Zeroes between two numbers always count [107].
3. Zeroes to the right of the number and to the right of the decimal point always count [16.50 and 0.1650].
4. Zeroes to the right of the number and to the left of the decimal point mayor may not count; this must be specified by the user [16,500].