Reminders 04-14-08:
-Next Exam April 16, Chapters 33-36

Outline:
-Diffraction Examples
-Blackbody Radiation
-Photoelectric Effect

$\phi = 2\pi m$ peaks in grating

adjacent minima occur at

$$\phi = 2\pi m + \frac{2\pi}{N}$$

$$\phi = \frac{2\pi d \sin\theta}{\lambda} \; ; \; d\phi = \frac{2\pi d \cos\theta d\theta}{\lambda}$$

$$d\phi = \frac{\cancel{2\pi} d \cos\theta d\theta}{\lambda} = \frac{\cancel{2\pi}}{N}$$

$$\frac{d \cos\theta \, d\theta}{\lambda} = \frac{1}{N}$$

$$d \sin\theta = m\lambda$$

$$d \cos\theta \, d\theta = m \, d\lambda$$

$$\frac{m \, d\lambda}{\lambda} = \frac{1}{N} \; ; \; \frac{d\lambda}{\lambda} = \frac{1}{mN}$$

$$\frac{\lambda}{d\lambda} = mN$$

- The full width of a 3 cm grating is illuminated by a sodium discharge tube. The lines in the grating are uniformly spaced at 775.0 nm. What is the angular separation in the first-order spectrum between the two wavelengths forming the sodium doublet (λ_1 = 589.0 nm, λ_2 = 589.6 nm).

$d \sin\theta = m\lambda \qquad \sin\theta = \frac{m\lambda}{d}$

find θ for each λ

$$\theta = \sin^{-1}\left[(1)\frac{589.0}{775}\right] = 49.464^\circ$$

$$\theta = \sin^{-1}\left[(1)\frac{589.6\,nm}{775}\right] = 49.532^\circ$$

$$\Delta\theta = \underline{.068^\circ}$$

A diffraction grating is illuminated with yellow light at normal incidence. The pattern seen on the screen behind the grating consists of three yellow spots, one at zero degrees and one each at ±45°. You now add red light of equal intensity coming in the same direction as the yellow light. The new pattern consists of

a. red spots at 0° and ± 45°.
b. yellow spots at 0° and ± 45°.
c. orange spots at 0° and ± 45°.
d. an orange spot at 0°, yellow at ± 45°, and red farther out.
e. an orange spot at 0°, yellow at ± 45°, and red closer in.

$$\sin\theta = \frac{m\lambda}{d}$$

- **Sunlight is incident on a diffraction grating which has 2750 lines/cm. The second-order spectrum over the visible range (400–700 nm) is to be limited to 1.75 cm along a screen a distance L from the grating. What is the required value of L?**

$$d \sin\theta = m\lambda$$

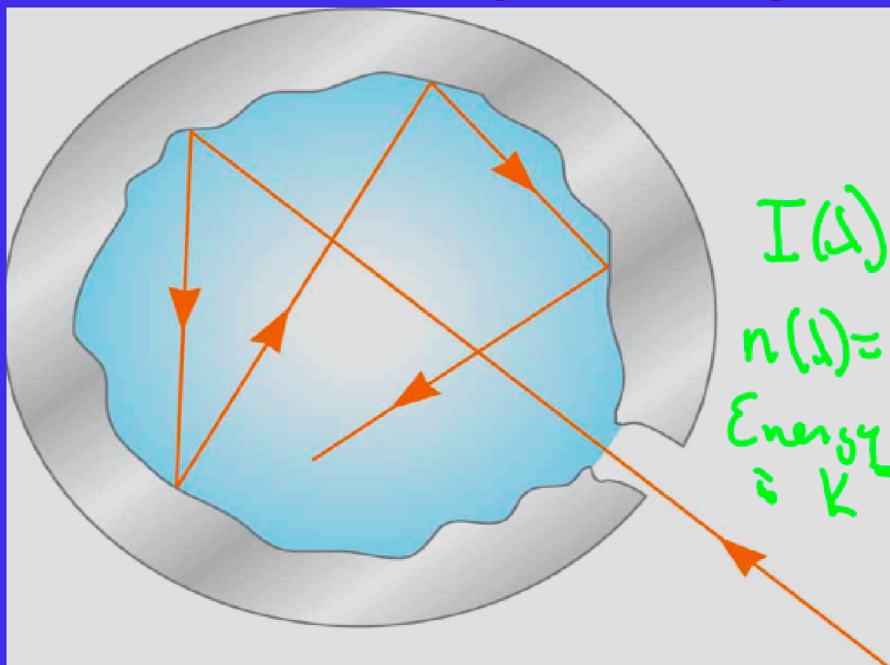
$$y = L\tan\theta \cong L\sin\theta = L\frac{m\lambda}{d}$$

Answer: 10.0cm

- An American standard television picture is composed of about 485 horizontal lines of varying light intensity. Assume that your ability to resolve the lines is limited only by the Rayleigh criterion and that the pupils of your eyes are 5 mm in diameter. Calculate the ratio of minimum viewing distance to the vertical dimension of the picture (50 cm) such that you will not be able to resolve the lines. Assume that the average wavelength of light coming from the screen is 550 nm.

Answer: 7.69m

Blackbody Cavity



$I(\lambda)$

$I(\lambda) = \frac{c}{4} u(\lambda)$

$n(\lambda) = 8\pi \lambda^{-4}$

Energy/oscillator $= kT$

System is designed so that incident radiation has little chance of being reflected out before being absorbed by cavity walls. Goal calculate the energy density of EM waves in cavity.

$$u(\lambda) = 8\pi \lambda^{-4} kT$$

$$I(\lambda) = \frac{c}{4} 8\pi \lambda^{-4} kT = 2\pi kTc\lambda^{-4}$$

$$I = \int_0^\infty I(\lambda)\,d\lambda = ?$$

Something's wrong!

- **Show that in the limit of long wavelengths Planck's result agrees with the Rayleigh-Jeans result (hint use $e^x=1+x+x^2+\ldots$ for $x<<1$)**

$$I(\lambda) = \frac{2\pi h c^2}{\left(e^{hc/\lambda kT} - 1\right)\lambda^5}$$

$$= \frac{2\pi h c^2}{\left(1 + \frac{hc}{\lambda kT} - 1\right)\lambda^{5}}$$

$$= \frac{2\pi \cancel{h} c^{\cancel{2}}}{\left(\cancel{h}c/\lambda kT\right)\lambda^{\cancel{5}\,4}}$$

$$= 2\pi kTc\,\lambda^{-4}$$

- **Show that the total energy density in a blackbody is proportional to T^4 in agreement with Stefan-Boltzmann's law (hint-when setting up the integral, substitute $x=hc/\lambda kT$; FYI, the value of the integral you obtain is $\pi^4/15$).**

$$I = \int_0^\infty I(\lambda)\, d\lambda$$

$$= \int_0^\infty \frac{2\pi h c^2 \, d\lambda}{\lambda^5 \left(e^{hc/\lambda kT} - 1\right)}$$

let $x = \dfrac{hc}{\lambda kT}$ $\qquad dx = -\dfrac{hc}{\lambda^2 kT}\, d\lambda$

$$d\lambda = -\lambda^2 \frac{kT}{hc}\, dx$$

$$= -\int_\infty^0 \frac{2\pi h c^2}{\lambda^3 (e^x - 1)} \cancel{\lambda^2} \frac{kT}{hc}\, dx$$

$$= \int_0^\infty \frac{2\pi hc^2}{\lambda^3 (e^x - 1)} \left(\frac{kT}{hc}\right)\left(\frac{hc}{kT}\right)^3 \left(\frac{kT}{hc}\right)^3 dx$$

$$= 2\pi hc^2 \left(\frac{kT}{hc}\right)^4 \int_0^\infty \frac{x^3\,dx}{e^x - 1}$$

$$= 2\pi hc^2 \left(\frac{kT}{hc}\right)^4 \frac{\pi^4}{15}$$

$$= \left[\frac{2}{15}\pi^5 hc^2 \left(\frac{k}{hc}\right)^4\right] T^4$$

$$\underline{5.67\times 10^{-8}\ T^4} \quad \text{Q.E.D.}$$

what happens as $\lambda \to 0$

$$I(\lambda) = \frac{2\pi h c^2}{\left(e^{hc/\lambda kT} - 1\right)\lambda^5}$$

$$\approx \frac{2\pi h c^2}{e^{hc/\lambda kT}\lambda^5}$$

$$= 2\pi h c^2 e^{-hc/\lambda kT}\lambda^{-5}$$

$I(\lambda) \to 0$ as $\lambda \to 0$