Reminders 02-27-08:

-Turn in Spring-Mass Worksheet Today. Change mass to 816 grams. Go to Web Page for new version.
-Ch. 15 Homework 3/2
-POW 4 by 5PM; remember these are extra credit problems.

Outline:

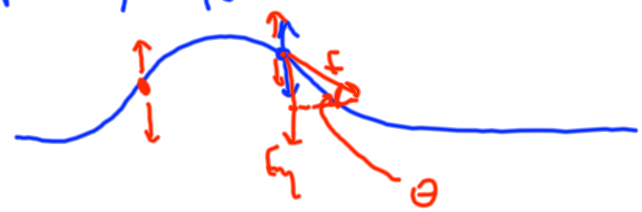
-Waves on a String
-Traveling Harmonic Waves
-Energy and Power of Waves
-Behavior of Waves
-Normal Modes

Why are the highest pitched strings on a musical instrument most likely to break first?

A rope of mass m hangs vertically from a ceiling. A transverse wave propagates up the rope. What happens to its speed?

$$v = f\lambda = \sqrt{\frac{T}{\mu}}$$

$P = \vec{F}\cdot\vec{v}$



$F_y = F\sin\theta$

let θ be small

$\sin\theta \approx \tan\theta = \frac{dy}{dx}$

$F_y = F\frac{dy}{dx}$

Power $= -F\frac{dy}{dx}v$

$y = -A\cos(kx-\omega t)$

$v = (+A\omega\sin(kx-\omega t))$

$\frac{dy}{dx} = -Ak\sin(kx-\omega t)$

$P = -F(-Ak\sin(kx-\omega t))[+A\omega\sin(kx-\omega t)]$

$= +FA^2k\omega\,\sin^2(kx-\omega t)$

$\frac{1}{2\pi}\int_0^{2\pi}\sin^2\theta\,d\theta = \left(\frac{x}{2} - \underline{\sin 2\theta}\right)\frac{1}{2\pi}$

$P_{avg} = +\frac{FA^2k\omega}{2}$ $\quad F = \mu v^2$

$= +\frac{\mu v^2 A^2 k\omega}{2}$; $\quad k = \frac{\omega}{v}$

$= +\frac{\mu v^2 A^2\left(\frac{\omega}{v}\right)\omega}{2} = +\frac{\mu v A^2\omega^2}{2}$

- Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string that has a linear mass density of $4.00 10^{-2}$kg/m. If the source can deliver a maximum power of 300 W and the string is under a tension of 100 N, what is the highest vibrational frequency at which the source can operate?

$$P_{avg} = \frac{1}{2}\mu\omega^2 A^2 v \qquad v = \sqrt{\frac{100}{.04001}}$$

$$P_{max} = \mu\omega^2 A^2 v$$

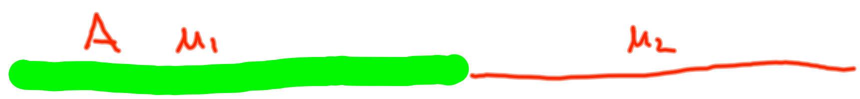
$$\omega = \sqrt{\frac{P_{max}}{\mu A^2 v}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{P_{max}}{\mu A^2 v}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{300}{(.04001)(.05)^2\sqrt{\frac{100}{.0401}}}}$$

$$f = 39\,Hz$$

- Two wires of different linear mass densities are soldered together end to end and are stretched under a tension F. The wave speed in the second wire is three time the first. When a harmonic wave traveling in the first wire is reflected at the junction of the wires the reflected wave has half the amplitude of the incident wave.
  - If the amplitude of the incident wave is A, what is the amplitude of the transmitted wave?
  - Assuming no energy loss in the wire, what fraction of the incident power is reflected and transmitted?



$$P_{avg} = \frac{1}{2} \mu A^2 \omega^2 v$$

$$P_o = P_r + P_t$$

$$\cancel{\tfrac{1}{2}} \mu_1 A^2 \cancel{\omega^2} v_1 = \cancel{\tfrac{1}{2}} \mu_1 A_r^2 \cancel{\omega^2} v_1 + \cancel{\tfrac{1}{2}} \mu_2 A_t^2 \cancel{\omega^2} v_2$$

$$\mu_1 A^2 v_1 = \mu_1 A_r^2 v_1 + \mu_2 A_t^2 v_2$$

$$v_2 = 3 v_1$$

$$A_r = \frac{1}{2} A$$

$$\mu_1 A^2 v_1 = \mu_1 \frac{A^2}{4} v_1 + \mu_2 A_t^2 \, 3 v_1$$

$$\frac{\cancel{3}}{4} \mu_1 A^2 \cancel{v_1} = \mu_2 A_t^2 \cancel{3} \cancel{v_1}$$

$$\frac{1}{4} \mu_1 A^2 = \mu_2 A_t^2$$

$$v = \sqrt{\frac{F}{\mu}}$$

$$\frac{9}{4} A^2 = A_t^2$$

$$A_t = \frac{3}{2} A$$

$$P_t = \frac{1}{2}\mu_2 A_t^2 \omega^2 v_2$$

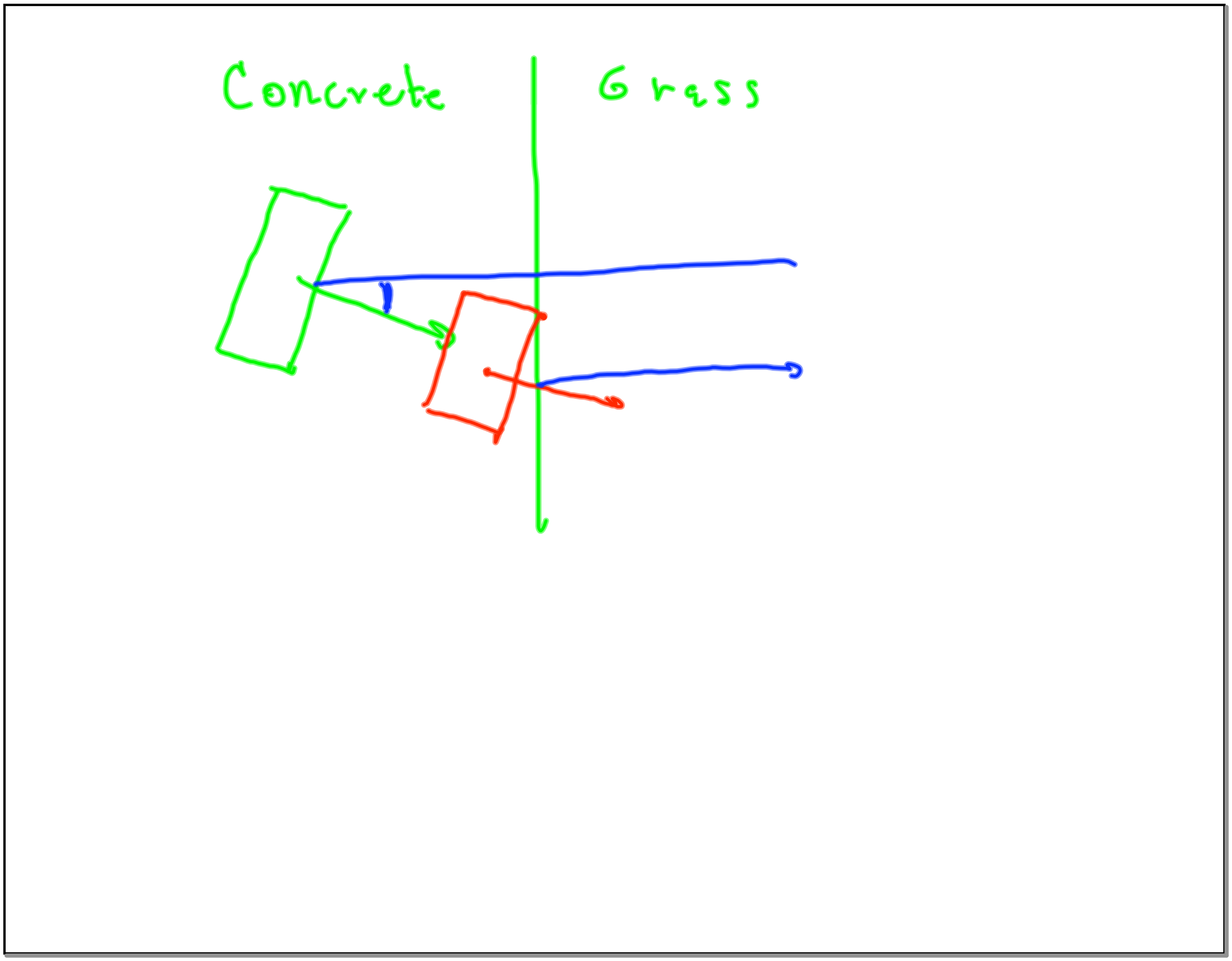
$$= \frac{1}{2}\left(\frac{\mu_1}{4}\right)\left(\frac{3}{2}A\right)^2 \omega^2 3v_1$$

$$= \frac{1}{2}\mu_1 A^2 \omega^2 v_1 \left(\frac{3}{4}\right)$$

$$P_0 = \frac{1}{2}\mu_1 A^2 \omega^2 v_1$$

$$P_t = \frac{3}{4}P_0$$

$$P_r = \frac{1}{4}P_0$$


Concrete
Grass

- A string that has a linear mass density of 0.040kg/m is under tension of 360N and is fixed at both ends. One of its resonance frequencies is 375 Hz. The next highest resonance frequency is 450Hz. What is the fundamental frequency? Which harmonics have the given frequencies? What is the length of the string?

Ans: f_1=75Hz; 5th and 6th harmonics; 2.0m