

Reminders 02-25-08:

- Turn in Spring-Mass Worksheet Wednesday. Change mass to 816 grams. Go to Web Page for new version.
- Ch. 13 Homework Due Tomorrow Night
- POW 4 by 5PM Wednesday; remember these are extra credit problems.

Outline:

- More SHM Examples
- Physical Pendulum
- Properties of Mechanical Waves
- Traveling Harmonic Waves

### Example

- A uniform rod of mass  $M$  and length  $L$  is pivoted about a point that is  $L/4$  above its center of mass so that it is free to rotate in a vertical plane. Show that the period of oscillation small angles is

$$T = 2\pi \sqrt{\frac{7L}{12g}}$$




Diagram: A horizontal line represents the pivot. A red rod is shown below it, pivoted at a point. A vertical line from the pivot to the center of mass is labeled  $L/4$ .

$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

$$d = \frac{L}{4}$$

$$I = I_{cm} + m\left(\frac{L}{4}\right)^2$$

$$= \frac{1}{12} mL^2 + m\frac{L^2}{16} = \frac{7}{48} mL^2$$

$$T = 2\pi \sqrt{\frac{\frac{7}{48} mL^2}{\frac{5}{4} \frac{L}{4}}} = 2\pi \sqrt{\frac{28}{48} \frac{L}{5}}$$

$$T = 2\pi \sqrt{\frac{7}{12} \frac{L}{g}}$$

$$y = \frac{6}{(x-4.5t)^2 + 3}$$

How do I prove that  $y$  satisfies the wave equation?

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

### Example

- A wave pulse is a function of  $x$  as described by the following equation:

$$y = \frac{6}{x^2 + 3}$$

If this wave is now moving with a  $v_x = 4.5$  m/s, what is  $y$  as a function of both  $x$  and  $t$ ?

- The equation of a wave is

$$y = 0.05 \sin\left[\frac{\pi}{2}(10x - 40t) - \frac{\pi}{4}\right] \text{ m}$$

- Find:  $\lambda$ ,  $f$ ,  $v$ , and the particle velocity for the wave at  $x=1.0\text{m}$ ,  $t=0.20\text{s}$

$$y = .05 \sin\left[5\pi x - 20\pi t - \frac{\pi}{4}\right]$$

$$k = 5\pi = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi}{5\pi} = \frac{2}{5} \text{ m} = 0.4 \text{ m}$$

$$v_y = \frac{dy}{dt} = (0.05) 20\pi \cos\left[5\pi x - 20\pi t - \frac{\pi}{4}\right]$$

$$= \pi \cos\left[5\pi x - 20\pi t - \frac{\pi}{4}\right]$$

$$= -\pi \cos\left[5\pi - 4\pi - \frac{\pi}{4}\right]$$


$$= -\pi \cos\left[\frac{3\pi}{4}\right]$$

$$= \pi \frac{\sqrt{2}}{2}$$

## Energy and Power

$$y = A \cos(kx - \omega t)$$

$\rightarrow m = \mu \Delta x$



$$\Delta K = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \mu \Delta x v^2$$

$$\frac{dy}{dt} = -A\omega \sin(kx - \omega t)$$

$$\left(\frac{dy}{dt}\right)^2 = v^2 = A^2 \omega^2 \sin^2(kx - \omega t)$$

$$\Delta K = \frac{1}{2} \mu \Delta x A^2 \omega^2 \sin^2(kx - \omega t)$$

$$\Delta U = \frac{1}{2} F \left(\frac{dy}{dx}\right)^2 \Delta x$$

$$\Delta U = \frac{1}{2} F [A k \sin(kx - \omega t)]^2 \Delta x$$

$$= \frac{1}{2} F A^2 k^2 \sin^2(kx - \omega t) \Delta x$$

$$= \frac{1}{2} \mu \omega^2 A^2 \sin^2(kx - \omega t) \Delta x$$

$$\Delta U + \Delta K = \frac{1}{2} \mu \Delta x A^2 \omega^2 \sin^2(kx - \omega t)$$

$$+ \frac{1}{2} A^2 \mu \omega^2 \sin^2(kx - \omega t) \Delta x$$

$$m = \sqrt{\frac{E}{\mu}} \quad m^2 = \frac{E}{\mu} \quad F = \mu v^2 = \left[ F = \mu \left(\frac{\omega}{k}\right)^2 \right]$$

$$\omega = kv$$

$$v = \frac{\omega}{k}$$

$$\Delta U + \Delta K = \mu \Delta x A^2 \omega^2 \sin^2(kx - \omega t)$$

$$= \Delta E$$

$$P = \frac{\Delta E}{\Delta t} = \mu \frac{\Delta x}{\Delta t} A^2 \omega^2 \sin^2(kx - \omega t)$$

$$P = \mu v A^2 \omega^2 \sin^2(kx - \omega t)$$

$$P_{avg} = \frac{\mu v A^2 \omega^2}{2}$$