Reminders 02-20-08:
-Turn in Spring-Mass Worksheet Monday
-Exam 1 Average 75.5%
-POW 4 Wed; remember these are extra credit problems.

Outline:
-Oscillatory Motion
-Simple Harmonic Motion
-Spring Mass System
-Pendulum
-Other Examples

$$x = A\cos\omega t + B\sin\omega t$$

$$= D\cos(\omega t + \phi)$$

$$\cos(\omega t + \phi) = \cos\omega t\cos\phi - \sin\omega t\sin\phi$$

$$x = D[\cos\omega t\cos\phi - \sin\omega t\sin\phi]$$

$$A = D\cos\phi \qquad B = -D\sin\phi$$

$$\frac{B}{A} = -\tan\phi$$

$$v = \frac{dx}{dt} = -A\omega\sin\omega t + B\omega\cos\omega t$$

$$x = A\cos\omega t + B\sin\omega t$$

$$v = \frac{dx}{dt} = -\underline{A\omega}\sin\omega t + B\omega\cos\omega t$$

$$|a_{max}| = A\omega^2$$

$$x = D\cos(\omega t + \phi)$$

$$\frac{dx}{dt} = -\underline{D\omega}\sin(\omega t + \phi) = -\underline{D\sqrt{\frac{k}{m}}}\sin(\omega t + \phi)$$

$$\frac{d^2x}{dt} = -D\omega^2\cos(\omega t + \phi)$$

$$|a_{max}| = D\omega^2$$

$$F = -kx = ma = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$x = A\cos\omega t + B\sin\omega t \quad or \quad x = D\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f \quad A = D\cos\phi \quad B = -D\sin\phi$$

The quantity ϕ is the <u>phase angle</u>; it tells you at what part of the cycle the object is at $t=0$. A & B or D & ϕ depend on initial conditions. If $v=0$ at $t=0$, then $B=0$ ($\phi=0$). If $x=0$ at $t=0$, $A=0$ ($\phi=\pi$) .

Suppose a 1kg mass on a horizontal surface is connected to a spring. Its period and amplitude of oscillation is 3.00s and 4.0cm, respectively. Assume v=0 at t=0s.

- **Write x=x(t),v=v(t), and a=a(t)**
- **Find t, when x=A/2 and -A/2.**
- **When is a= zero the first time?**
- **When does v reach a first maximum?**
- **How do you determine k?**
- **When is its potential energy equal to its kinetic energy?**

$x = A\cos\omega t + B\sin\omega t$

$B = 0$ since $v = 0$ at $t = 0$

$A = 4.0$ cm

$\omega = \frac{2\pi}{T} = \frac{2\pi}{3}$

$\longrightarrow x = (4.0\text{cm})\cos\frac{2\pi}{3}t$

$v = -(4.0\text{ cm})\left(\frac{2\pi}{3}\right)\sin\frac{2\pi}{3}t$

$a = -(4.0\text{ cm})\left(\frac{2\pi}{3}\right)^2\cos\left(\frac{2\pi}{3}t\right)$

$2\text{ cm} = 4\text{ cm} \cos \frac{2\pi}{3} t$

$\frac{1}{2} = \cos \frac{2\pi}{3} t$

$\cos^{-1} \frac{1}{2} = \frac{2\pi}{3} t$

$\frac{\pi}{3} = \frac{2\pi}{3}$ $\quad t = \underline{1/2 \text{ s}}$

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} t$

$\vec{v}$ reaches 1st max at $\frac{3}{4}$ s

want $\cos \frac{2\pi}{3} t = 0$ or

$\sin \frac{2\pi}{3} t = 1$

$\frac{2\pi}{3} t = \frac{\pi}{2}$

$t = \underline{\frac{3}{4} \text{ s}}$ 1st time

$\vec{a} = 0$

$$\omega = \sqrt{\frac{k}{m}} \qquad \omega^2 = \frac{k}{m}$$

$$\left(\frac{2\pi}{3}\right)^2 = \frac{k}{1}$$

$$k = (1)\left(\frac{2\pi}{3}\right)^2$$

$$\text{Total energy} = \frac{1}{2} k A^2$$

$$U = K$$

$$\frac{1}{2}\left(\cancel{\frac{1}{2}}\cancel{k} A^2\right) = \cancel{\frac{1}{2}}\cancel{k} x^2$$

$$\frac{1}{2} A^2 = x^2$$

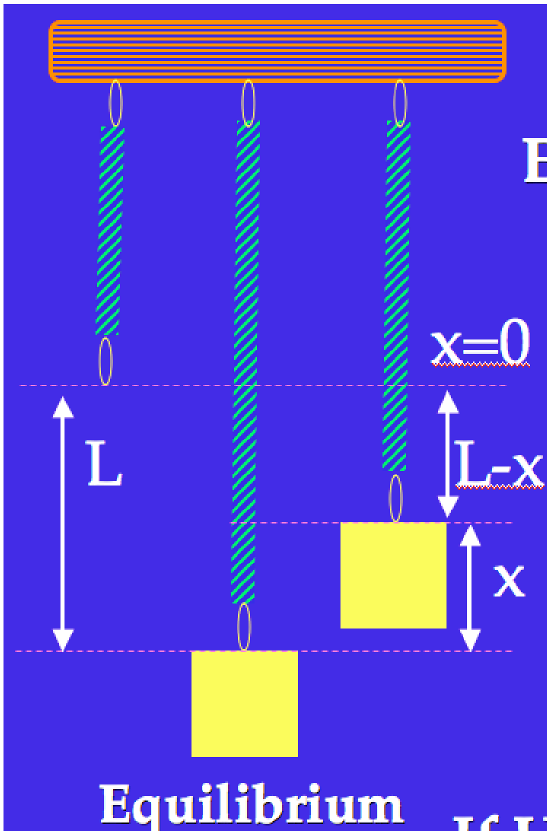
$$x = \frac{A}{\sqrt{2}}$$

$$x = \underline{4} \cos\left(\frac{2\pi}{3} t\right)$$

$$\frac{4}{\sqrt{2}} = 4 \cos\left(\frac{2\pi}{3} t\right)$$

$$\frac{\cancel{\pi}}{4} = \frac{2\cancel{\pi}}{3} t$$

$$t = \frac{3}{8}\,s$$



Equilibrium position of mass

$$kL = mg \Rightarrow L = mg/k$$

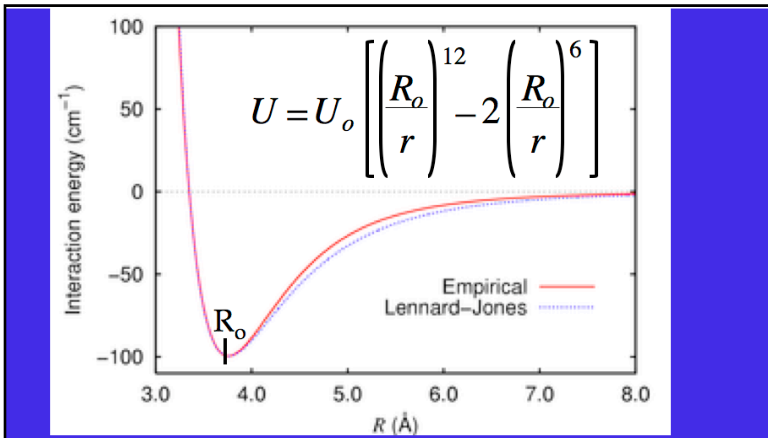
$$F_{net} = k(L - x) - mg = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Same as before !!

If U_g=0 at equilibrium, then

$U=1/2k(L-x)^2+mgx=1/2kL^2+1/2kx^2$



$$\frac{dU}{dr} = U_o\left[-12\left(\frac{R_o^{12}}{r^{13}}\right) + 12\left(\frac{R_o^{6}}{r^{7}}\right)\right]$$

$$F = -\frac{dU}{dr} = U_o\left[12\left(\frac{R_o^{12}}{r^{13}}\right) - 12\left(\frac{R_o^{6}}{r^{7}}\right)\right]$$

let $r = R_o + x$

$$F = 12U_o\left[\frac{R_o^{12}}{(R_o+x)^{13}} - \frac{R_o^{6}}{(R_o+x)^{7}}\right]$$

$$= 12U_o\left[\frac{R_o^{12}}{R_o^{13}\left(1+\frac{x}{R_o}\right)^{13}} - \frac{R_o^{6}}{R_o^{7}\left(1+\frac{x}{R_o}\right)^{7}}\right]$$

$$= \frac{12U_o}{R_o}\left[\frac{1}{\left(1+\frac{x}{R_o}\right)^{13}} - \frac{1}{\left(1+\frac{x}{R_o}\right)^{7}}\right]$$

Use binomial expansion

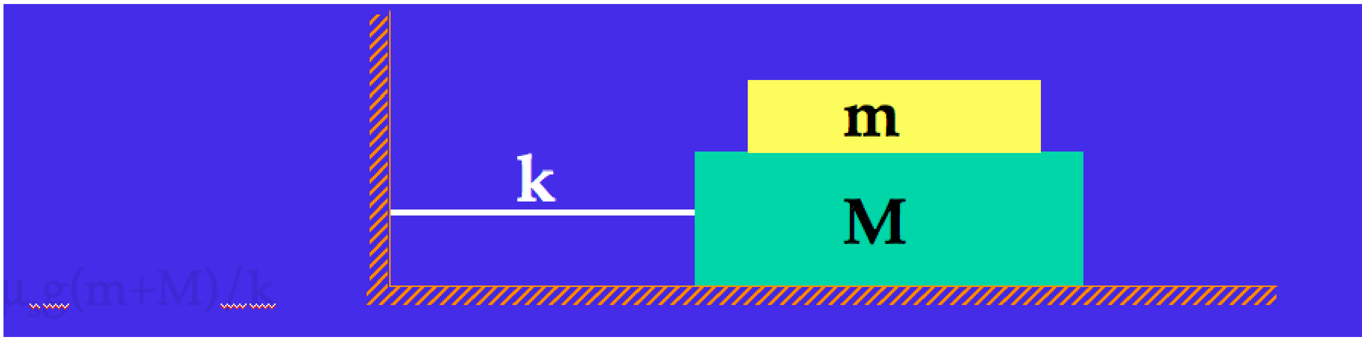
$$(1+x)^n \cong \underline{1 + nx} + \frac{n(n-1)}{2!}x + \ldots$$

$$F = \frac{12U_o}{R_o}\left[1 - 13\frac{x}{R_o} - \left(1 - 7\frac{x}{R_o}\right)\right]$$

$$F = \frac{12U_o}{R_o}\left[-13\frac{x}{R_o} + \frac{7x}{R_o}\right] = -\left(\frac{72U_o}{R_o^2}\right)x$$

$$F = -\left(\frac{72U_o}{R_o^2}\right)x = -kx$$

SIMPLE HARMONIC Motion!



When m starts to slip

$$f_s = \mu_s mg = ma \qquad a = \mu_s g$$

$$-kx + f_s = -Ma$$

$$-kx + \mu_s mg = -M\mu_s g$$

$$+kx = +M\mu_s g + \mu_s mg$$

$$x = \frac{\mu_s g(m+M)}{k}$$

- A hole is drilled along the diameter of the Earth. A ball is dropped into this hole. Describe its motion and derive an expression for the acceleration and the position of ball in this hole. In addition, discuss its motion.

- A uniform rod of mass M and length L is pivoted about a point that is L/4 above its center of mass so that it is free to rotate in a vertical plane. Show that the period of oscillation small angles is

$$T = 2\pi\sqrt{\frac{7L}{12g}}$$