Reminders 05-06-10:

-POW 12 Due May 11

-If you are transferring after this semester please have picture for our Hall of Fame.

Objectives

-Kepler's Laws

-Orbit Problem involving Conservation of L

-Gravitational Field for Continuous Mass Distributions

A rocket is fired at 60° to the local vertical with an initial speed $v_0 = (GM/R)^{1/2}$, where M is the mass of the earth and R is its radius. What is its maximum distance from the earth's center?

Conserve L $\qquad L_i = L_f$

$$\cancel{m} v_0 R_E \sin 60° = \cancel{m} v_f r_f$$

Conserve E

$$\frac{1}{2} m v_0^2 - \frac{G M_e m}{R_E} = \frac{1}{2} m v_f^2 - \frac{G M_e m}{r_f}$$

$$v_f = \frac{v_0 R_e \sin 60}{r_f}$$

$$\frac{1}{2} \cancel{m} v_0^2 - \frac{G M_e \cancel{m}}{R_e} = \frac{1}{2} \cancel{m} \left[\frac{v_0 R_e \sin 60}{r_f}\right]^2 - \frac{G M_e \cancel{m}}{r_f}$$

$$v_0^2 - \frac{2 G M_e}{R_e} = \frac{v_0^2 R_e^2 \sin^2 60}{r_f^2} - \frac{2 G M_e}{r_f}$$

$$\frac{\cancel{GM_e}}{R_e} - \frac{2 \cancel{GM_e}}{R_e} = \frac{\cancel{GM_e}}{R_e} \frac{R_e^2 \sin^2 60}{r_f^2} - \frac{2 \cancel{GM_e}}{r_f}$$

$$-\frac{1}{R_e} = \frac{R_e \sin^2 60}{r_f^2} - \frac{2}{r_f}$$

$$-\frac{r_f^2}{R_e} = R_e \sin^2 60 - 2 r_f$$

$$-r_f^2 = R_e^2 \sin^2 60 - 2 r_f R_e$$

$$0 = R_e^2 \sin^2 60 - 2 r_f R_e + r_f^2$$

$$r_f = \frac{2R_e \pm \sqrt{4R_E^2 - 4(R_e^2 \sin^2 60)}}{2}$$

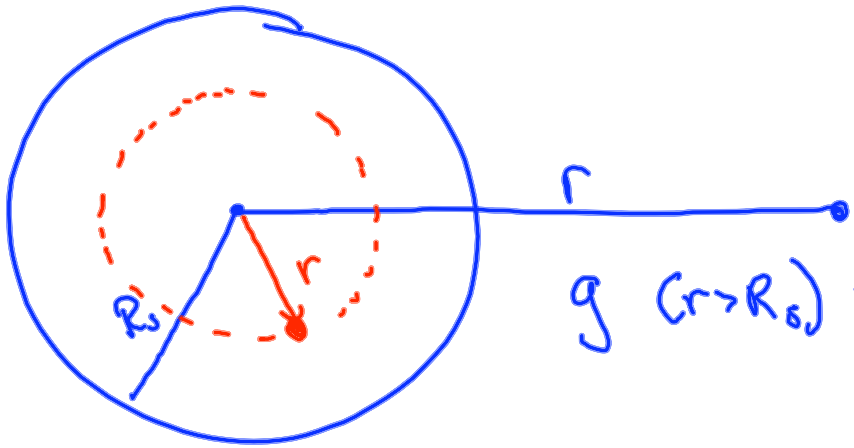
$$= \frac{2R_e \pm 2R_e\sqrt{1 - \sin^2 60}}{2}$$

$$= R_e (1 \pm \cos 60) \quad \text{choose } (+)$$

$$r_f = \frac{3}{2} R_e$$

Title: May 6-11:20 AM (2 of 4)

What is the field strength inside a homogeneous sphere of total mass M and radius R.

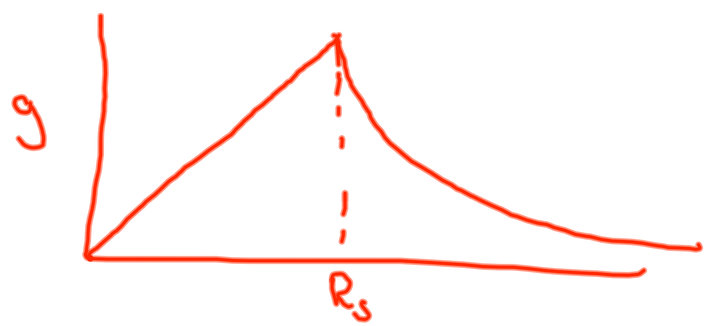


$$g\,(r>R_s) = \frac{GM}{r^2}$$

$$g_{solid\,sphere} = \frac{G M_{enclosed}}{r^2}$$

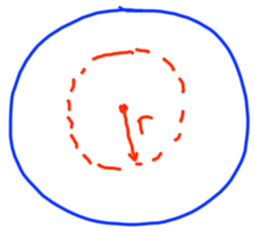
$$M_{enclosed} = \rho\, V_{enclosed} = \frac{M}{\cancel{\tfrac{4}{3}\pi} R_s^3}\,\cancel{\tfrac{4}{3}\pi}\, r^3 = \frac{M r^3}{R_s^3}$$

$$g = \frac{G M r^3}{r^2 R_s^3} = \frac{G M r}{R_s^3}$$



What if the density of the sphere varies linearly with the radius of the sphere?

$\rho = Cr$ total mass is M & Radius is R_s



$$g_{ss} = \frac{G M_{enclosed}}{r^2}$$

$$M_{enclosed} = \int \rho \, dV$$

$$dV = 4\pi r^2 dr$$

$$dm = Cr \, 4\pi r^2 dr$$

$$M_{enclosed} = \int_0^r 4\pi c r'^3 dr' = \pi C r^4$$

Need C

$$M = \int_0^{R_s} 4\pi c r'^3 dr = \pi C R_s^4$$

$$C = \frac{M}{\pi R_s^4}$$

$$M_{enclosed} = \pi \left(\frac{M}{\pi R_s^4}\right) r^4 = \frac{M r^4}{R_s^4}$$

$$g = G\left[\frac{M r^4}{R_s^4}\right]\frac{1}{r^2} = \frac{G M r^2}{R_s^4}$$