Reminders 05-04-10:

-POW 12 Due May 11

-Exam 4 Average 59%

-Quiz in Recitation Thursday in Lecture Time Permitting

-If you are transferring after this semester please have your picture for our Hall of Fame.

Objectives

-Gravitational Field

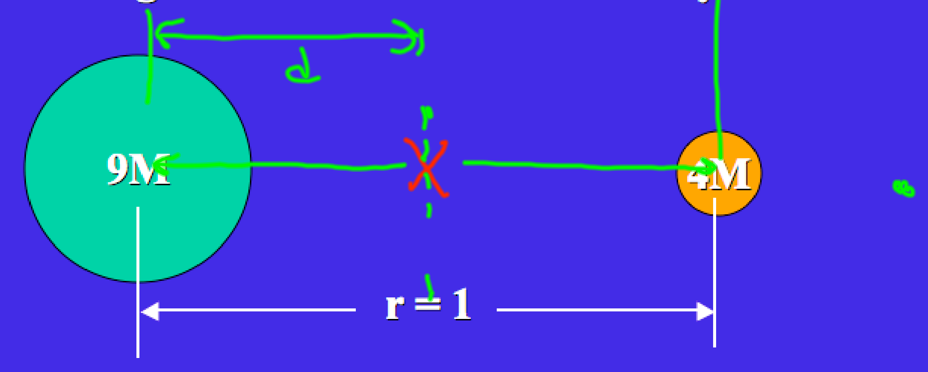
-Gravitational Potential Energy

-Conservation of Energy

-Orbit Problems

-Kepler's Laws

For the situation shown in the sketch, determine where the gravitational field is identically zero.




$$\frac{\cancel{G}9\cancel{M}}{d^2} = \frac{\cancel{G}4\cancel{M}}{(1-d)^2}$$

$$\frac{9}{d^2} = \frac{4}{(1-d)^2}$$

$$9(1-d)^2 = 4d^2$$

$$3(1-d) = 2d$$

$$3 - 3d = 2d$$

$$3 = 5d$$

$$d = \frac{3}{5}$$

$W = \int \vec{F} \cdot d\vec{r} \qquad \vec{F} = -\frac{GM_1M_2}{r^2}\hat{r}$

$d\vec{r}$ component along $\hat{r}$
another component $\perp$ to $\hat{r}$

$$\vec{F} \cdot d\vec{r} = \frac{-GM_1M_2}{r^2}\,dr$$

$$W_g = \int_{r_a}^{r_b} -\frac{GM_1M_2}{r^2}\,dr = \frac{GM_1M_2}{r}\Big|_{r_A}^{r_B}$$

$$W_g = \frac{GM_1M_2}{r_b} - \frac{GM_1M_2}{r_a}$$

$W_g$ is indep of path

Force is conservative.

Define $U$.

$W_g = -\Delta U$

$$\Delta U = U_B - U_A = \frac{GM_1M_2}{r_a} - \frac{GM_1M_2}{r_B}$$

let $r_a = \infty$

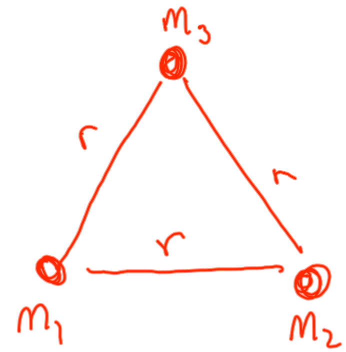
$$U_B - U_A = \frac{-GM_1M_2}{r_B}$$

$U_A = 0$ since $r_a = \infty$

$$U_B = \frac{-GM_1M_2}{r_B}$$

$$U = \frac{-GM_1M_2}{r}$$

Title: May 4-11:18 AM (3 of 7)




How much work is required to assemble this system of particles into the above configuration

Object 1 requires no work

Object 2 requires $-\dfrac{GM_1M_2}{r}$

to overcome field due to object 1.

Object 3 $\quad -\dfrac{GM_3M_1}{r} + -\dfrac{GM_3M_2}{r}$

$$W_{TOTAL} = -\left(\frac{GM_1M_2}{r} + \frac{GM_3M_1}{r} + \frac{GM_3M_2}{r}\right)$$

$$= \frac{1}{2}\sum_{i \neq j} \frac{GM_iM_j}{r_{ij}}$$

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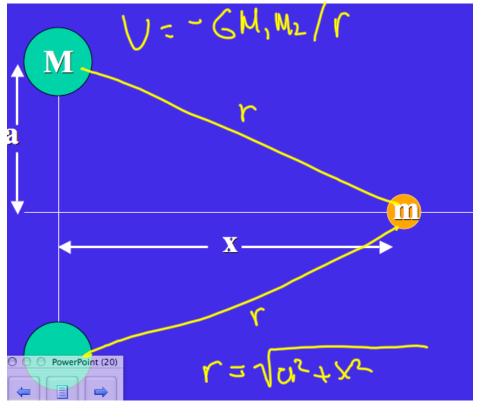
$$U = -\left(\frac{GM_1M_2}{r} + \frac{GM_3M_1}{r} + \frac{GM_3M_2}{r}\right)$$




$$U = \frac{-2GMm}{\sqrt{a^2+x^2}} \qquad F = -\frac{\partial U}{\partial x}$$

$$F = \frac{\partial}{\partial x}\frac{2GMm}{\sqrt{a^2+x^2}} = \frac{-2GMm\,x}{(a^2+x^2)^{3/2}}$$

where is force maximum

$$\frac{\partial F}{\partial x} = -2GMm\left[\frac{\partial}{\partial x}\frac{x}{(a^2+x^2)^{3/2}}\right] = 0$$

$$\frac{\partial}{\partial x}\, x(a^2+x^2)^{-3/2} = (a^2+x^2)^{-3/2} + -x(a^2+x^2)^{-5/2}\,3x$$

$$= (a^2+x^2)^{-3/2} - 3x^2(a^2+x^2)^{-5/2} = 0$$

$$\frac{1}{(a^2+x^2)^{3/2}} - \frac{3x^2}{(a^2+x^2)^{5/2}} = 0$$

$$\frac{(a^2+x^2) - 3x^2}{(a^2+x^2)^{5/2}} = 0$$ need num. to be zero.

$$a^2 - 2x^2 = 0 \qquad a^2 = 2x^2$$

$$x = \frac{a}{\sqrt{2}}$$

$$\vec{g} = -\frac{G m_1}{r^2}\,\hat{r} \qquad \vec{g} = \frac{\vec{F}}{m}$$

This expression allows one to determine the strength of the gravitational field at any location  space. The term $r^2$ is the distance from the source of the field to the point where the field is to be determined, squared.
The unit vector in the above equation is directed from the source of the field to the point at which the field is to be determined.

$$\vec{F} = -\frac{G m_1 m_2}{r^2}\hat{r}$$ Central force

$$W_c = \int -\frac{G m_1 m_2}{r^2}\hat{r}\cdot d\vec{r}$$

$$= \int_{r_a}^{r_b} -G\frac{m_1 m_2}{r^2}\,dr$$

$$\Delta U = -W_c = \int_{r_a}^{r_B} \frac{G m_1 m_2}{r^2}\,dr$$

let $r_b \to \infty$

$$U_A = -\frac{G m_1 m_2}{r_a}$$

$$W = \int_{r_a}^{r_b} f(|\vec{r}|)\,dr = f(r_b) - f(r_a)$$

$\Rightarrow$ force is conservative

$\Rightarrow$ we can define potential energy.

$$\Rightarrow \oint f(|\vec{r}|)\,dr = 0$$

as we learned in Chapter 8.