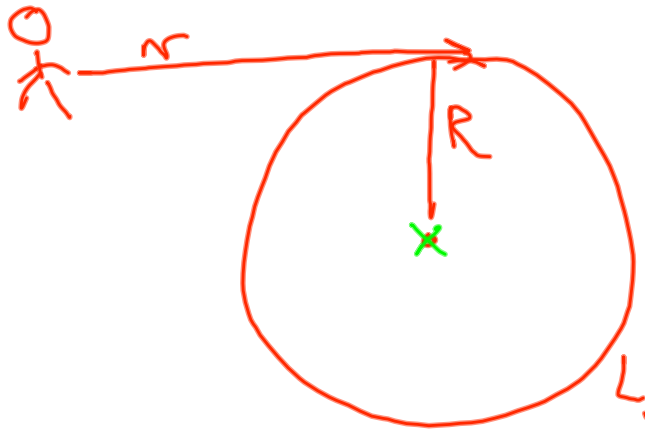
Reminders 04-20-10:

-POW 10 Due next Thursday
-Quiz Today
-Exam 4 April 29

Objectives

-Conservation of Angular Momentum
-Examples
-More About Chasles' Theorem

A 100 kg man runs at 2 m/s tangent to and then jumps onto the perimeter of a stationary merry-go-round that has a radius of 2 m and a mass of 50 kg. Treat the merry-go-round as a uniform disk ($I = 1/2MR^2$) and find the angular speed of the merry-go-round/man system.



$\vec{L}_i = \vec{r} \times \vec{p}$

$|\vec{L}_i| = Rmv$

$L_f = (I_p + I_d)\omega$

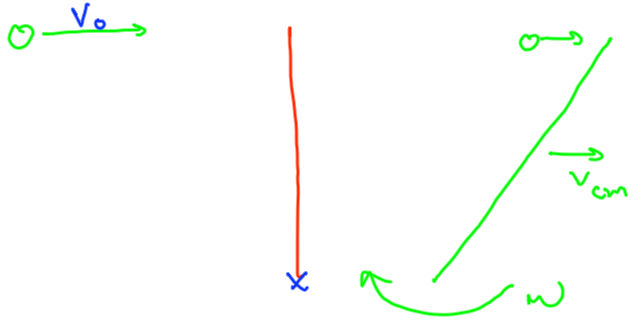
$L_i = L_f$

$mRv = \left(mR^2 + \frac{1}{2}MR^2\right)\omega$

$\omega = \dfrac{mRv}{mR^2 + \frac{1}{2}MR^2}$

$= \dfrac{mv}{mR + \frac{1}{2}MR} = 0.8/s$

A 1 kg ball traveling at 1 m/s perpendicular to a 1 kg thin rod 2 m in length that is at rest, undergoes an elastic collision with the end of the rod. What are the linear velocities of the ball and the rod and the angular speed of the rod after the collision? (Hint: the ball's path is unchanged after the collision.)



$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$$

$$mv_0 = mv_f + mv_{cm} \rightarrow v_0 = v_f + v_{cm}$$

$$-mv_0\ell = -mv_f\ell - I\omega \rightarrow \omega = \frac{mv_f\ell - mv_0\ell}{I}$$

$$\omega = \frac{mv_0\ell - mv_f\ell}{I}$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}m(v_0 - v_f)^2 + \frac{1}{2}I\left[\frac{mv_0\ell - mv_f\ell}{I}\right]^2$$

$$mv_0^2 = mv_f^2 + m(v_0 - v_f)^2 + \frac{m^2\ell^2}{I}(v_0 - v_f)^2$$

$$mv_0^2 = mv_f^2 + m(v_0 - v_f)^2 + \frac{m^2\ell^2}{\frac{1}{3}m\ell^2}(v_0 - v_f)^2$$

$$v_0^2 = v_f^2 + (v_0 - v_f)^2 + 3(v_0 - v_f)^2$$

$$v_0^2 = v_f^2 + 4(v_0 - v_f)^2$$

$$v_s^2 = v_f^2 + 4(v_0 - v_f)^2$$

$$v_0^2 - v_f^2 = 4(v_0 - v_f)^2$$

$$(v_0 + v_f)(v_0 - v_f) = 4(v_0 - v_f)$$

$$v_0 + v_f = 4(v_0 - v_f) = 4v_0 - 4v_f$$

$$5v_f = 3v_0$$

$$v_f = \frac{3}{5} v_0 \qquad v_f = \frac{3}{5}\ m/s$$

$$v_{cm} = v_0 - v_f$$

$$= 1 - \frac{3}{5} = \frac{2}{5}\ m/s$$

$$\omega = \frac{mv_0 l - mv_f l}{I} = \frac{ml(v_0 - v_f)}{\frac{1}{3} ml^2}$$

$$= \frac{3(v_0 - v_f)}{l} = \frac{\frac{3}{2}}{2} = \frac{3}{4}\ rad/s$$