Reminders 04-15-10:

-POW 10 Due next Thursday
-Chapter 10 and 11 quiz in class next Tuesday

Objectives

-Formal Definition of Torque
-Formal Definition of Angular Momentum
-Relation Between Torque and Angular Momentum

Example:

If a force F = (3 i + 2 k) N is applied at the position r = (1 i – 2 j) m what is the torque due to the force?

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 0 \\ 3 & 0 & 2 \end{vmatrix} = \hat{i}\begin{vmatrix} -2 & 0 \\ 0 & 2 \end{vmatrix} + \dots -$$

ith component is $-4\hat{i}$

$$-\hat{j}\begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix} = -2\hat{j}$$

$$\hat{k}\begin{vmatrix} 1 & -2 \\ 3 & 0 \end{vmatrix} = 6\hat{k}$$

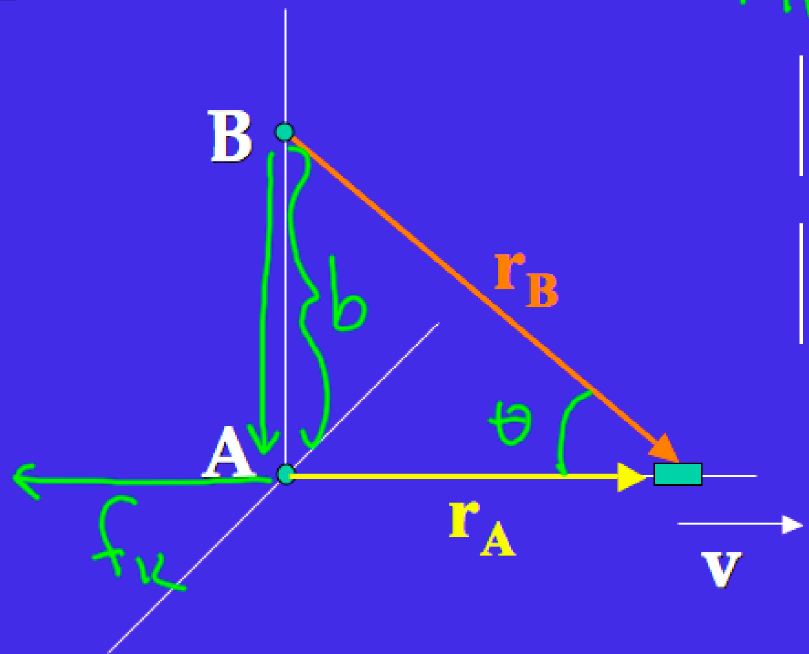
$$\boxed{\vec{\tau} = -4\hat{i} - 2\hat{j} + 6\hat{k}}$$

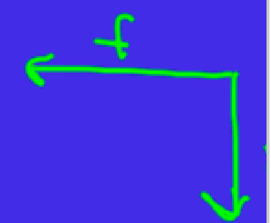
Consider a particle of mass m moving along t
The only force acting on it is kinetic friction.
at the torque about points A and B (a distanc
point A).

$|f_k||\vec{r}_b|\sin\theta$

$$\left|\sum \vec{\tau}_A\right| = |\vec{r}_A \times \vec{f}_k| =$$

$$\left|\sum \vec{\tau}_B\right| = |\vec{r}_B \times \vec{f}_k| =$$





Notice that the sum of the torques depends on
of origin. However, it can be shown that if th
forces on an object is zero then the sum of the
be the same regardless of the choice of origin.

Example

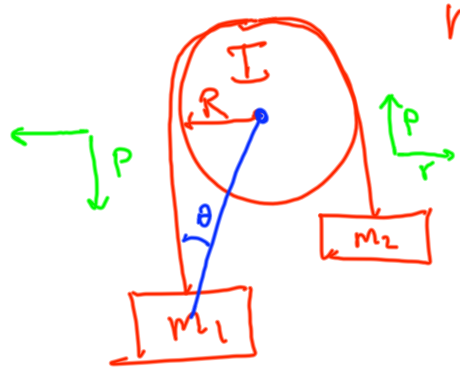
The position of a particle of mass 3 kg is given by $\vec{r} = (6\,\hat{i} + 8\,t\,\hat{j})$ m. Determine the angular momentum of the particle about the origin as a function of time.

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 8\hat{j}$$

$$\vec{L} = (6\hat{i} + 8t\hat{j}) \times (24\hat{j})$$

$$= 144\,\hat{k}\ \text{Kgm}^2/\text{s}$$



$m_1 > m_2$

$$L_1 = r m_1 v \sin\theta$$
$$= +m_1 v R$$
$$L_2 = +m_2 v R$$
$$L_p = +I\omega$$

$$\rightarrow L_{system} = (m_1+m_2) vR + I\omega$$
$$F_{ext} = m_1 g - m_2 g = (m_1-m_2) g$$
$$\tau_{ext} = (m_1-m_2) gR = \frac{dL}{dt}$$

$$\frac{dL_{system}}{dt} = (m_1+m_2) aR + I\alpha$$

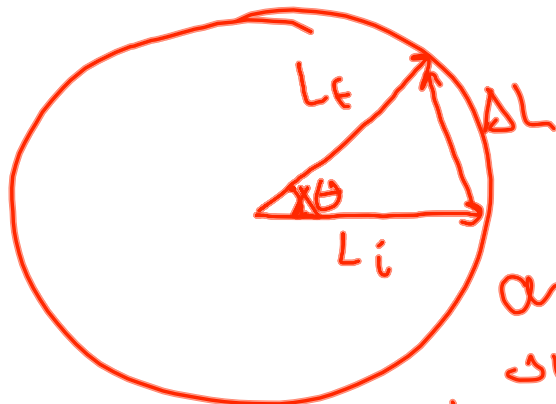
$$(m_1-m_2) g R = (m_1+m_2) aR + I\alpha$$
$$= (m_1+m_2) aR + I \frac{a}{R}$$

$$(m_1-m_2) g = \left[(m_1+m_2) + \frac{I}{R^2}\right] a$$

$$a = \frac{(m_1-m_2) g}{m_1+m_2 + I/R^2}$$

Calculate precessional freq. of gyroscope.

$|L_i| = |L_f| = I\omega$



Let $\Delta\theta$ be small

arc length swept out in time Δt is $L_i \Delta\theta$

arc length = ΔL for small $\Delta\theta$

$$\frac{\Delta L}{\Delta t} = L_i \frac{\Delta\theta}{\Delta t} \qquad \frac{\Delta L}{\Delta t} = I\omega\Omega = mgR$$

$$\Omega = \frac{mgR}{I\omega}$$

valid as long as $\omega \gg \Omega$