Reminders 04-13-10:

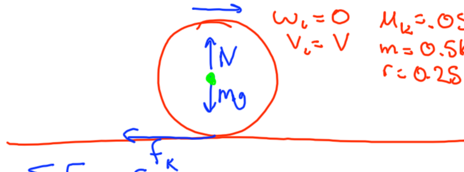
-POW 9 Due Today
-POW 10 Due next Thursday
-Chapter 10 and 11 quiz in class next Tuesday

Objectives

-Dynamics of Rolling Motion
-Conservation of Energy
-Formal Definition of Torque

$\sum \tau = I\alpha \qquad \sum F = ma_{cm}$

A "bowling ball" of mass 0.5 kg and diameter 0.5 m is given an initial velocity of 3 m/s and laid smoothly on a horizontal surface that has a coefficient of kinetic friction $\mu_K = 0.05$.
How far does it travel before it rolls without slipping?
How fast is it traveling at that point?



$\omega_i = 0$
$V_i = V$

$\mu_k = .05$
$m = 0.5kg$
$r = 0.25m$

$\sum F_x = f_k = ma_{cm} = \mu_k N =$

$\mu_k \cancel{m} g = \cancel{m} a_{cm}$

$a_{cm} = \mu_k g$

$V_f^2 - V_i^2 = 2a\Delta x$

$V_f^2 = V_i^2 + 2a\Delta x$

$\boxed{V_f^2 = V_i^2 - 2\mu g \Delta x \qquad V_f = V_i - \mu g t}$

$V_f = V_i + at$

Consider $\sum \tau$

$f_k R = I\alpha$

$\mu_k \cancel{m} g R = \frac{2}{5} \cancel{m} R^2 \alpha$

$\alpha = \frac{5}{2}\frac{g}{R}\mu_k$

$\omega_f = \omega_i + \alpha t \qquad -\omega_f = -\frac{5}{2}\frac{g}{R}\mu_k t$

When object starts to roll $V_{cm} = R\omega$

$-R\omega_f = -\frac{5}{2}\frac{g}{\cancel{R}}\mu_k t \cancel{R} = V_{cm} = V_f$

$+\frac{5}{2}\mu g t = V_i - \mu g t$

$\rightarrow V_i = \frac{7}{2}\mu g t \qquad t = \frac{V_i}{\frac{7}{2}\mu g} = \frac{2V_i}{7\mu g}$

$t = \frac{2V_i}{7\mu_k g} \qquad V_f = V_i - \mu g\left(\frac{2}{7}\frac{V_i}{\mu g}\right) = \frac{5}{7}V_i$

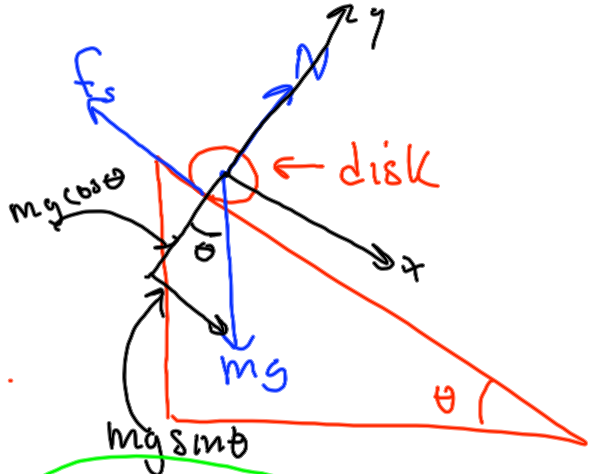
Bowling ball is traveling at $\frac{5}{7}v_i$ when it starts to roll.

$$\left(\frac{5}{7}v_i\right)^2 - v_i^2 = -2\mu g \Delta X$$

$$\left(\frac{25}{49} - 1\right)v_i^2 = -2\mu g \Delta X$$

$$\frac{24}{49}v_i^2 = 2\mu g \Delta X$$

$$\frac{12}{49}\frac{v_i^2}{\mu g} = \Delta X = \frac{12}{49}\frac{(3\,m/s)^2}{(.05)(9.8)} = \underline{4.5\,m}$$



$$\sum F_x = mg\sin\theta - f_s = ma_{cm}$$

$$\sum F_y = N - mg\cos\theta = 0$$

Rolling motion f_s not related to N.

Sum torques

$$f_s R = \frac{1}{2} mR^2 \alpha \qquad a_{cm} = R\alpha$$

$$\alpha = \frac{a_{cm}}{R}$$

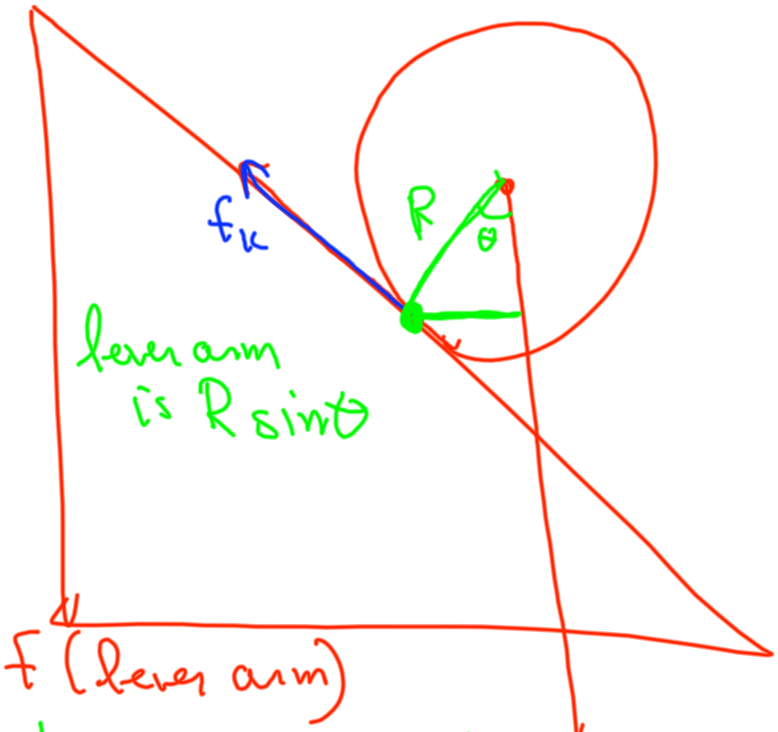
$$f_s R = \frac{1}{2} m R^2 \frac{a_{cm}}{R}$$

$$f_s = \frac{1}{2} m a_{cm}$$

$$mg\sin\theta - \frac{1}{2} m a_{cm} = m a_{cm}$$

$$mg\sin\theta = \frac{3}{2} m a_{cm}$$

$$a_{cm} = \frac{2}{3} g\sin\theta$$



$\tau = F(\text{lever arm})$

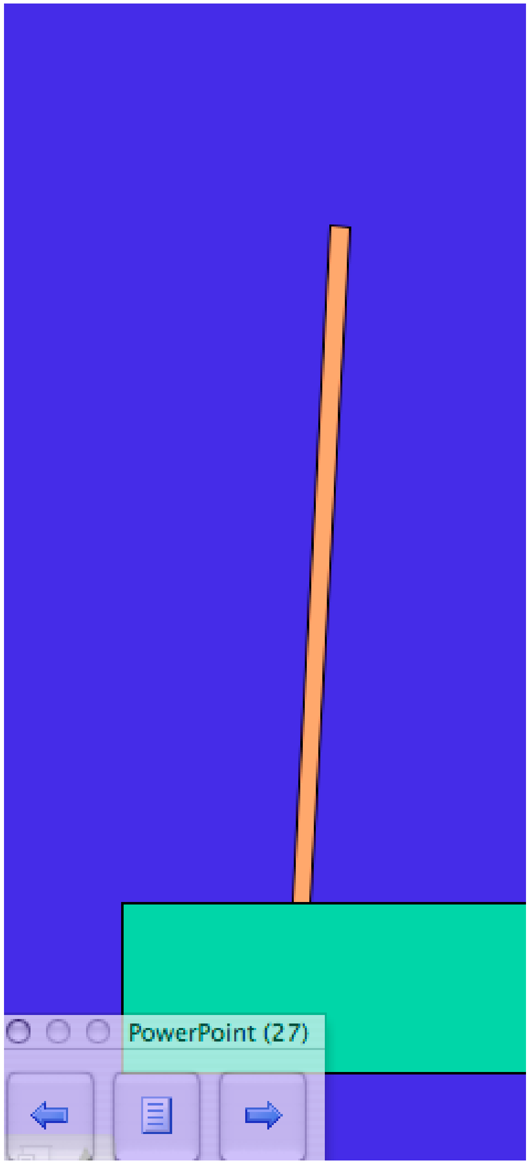
Sum torques about point of contact

$\rightarrow mgR\sin\theta = \left(\frac{1}{2}mR^2 + mR^2\right)\alpha$

$mgR\sin\theta = \frac{3}{2}mR^2\alpha$

$\cancel{m}g\cancel{R}\sin\theta = \frac{3}{2}\cancel{mR}a_{cm}$

$a_{cm} = \frac{2}{3}g\sin\theta$



$$\Delta U + \Delta K_{rot} = 0$$

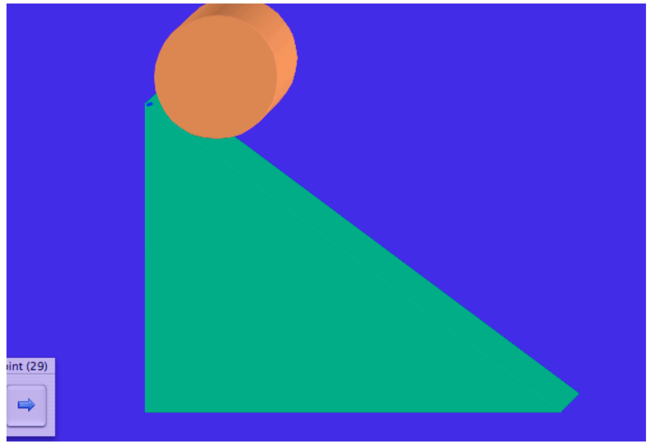
$$-mg\frac{L}{2} + \frac{1}{2} I \omega_f^2 = 0$$

$$mgL = I\omega^2$$

$$\cancel{m}g\cancel{L} = \frac{1}{3}\cancel{m}L^{\cancel{2}}\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}}$$

$$V_{end} = L\omega = L\sqrt{\frac{3g}{L}} = \sqrt{3gL}$$



$$\Delta U + \Delta K = 0$$

$$-mgh + \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 = 0$$

$$v_{cm} = r\omega$$

$$\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\left(\frac{v_{cm}}{r}\right)^2 = mgh$$

$$mv_{cm}^2 + I\frac{v^2}{r^2} = 2mgh$$

$$v_{cm}^2\left(m + \frac{I}{r^2}\right) = 2mgh$$

$$v = \sqrt{\frac{2mgh}{m + \frac{I}{r^2}}}$$

let $I = \frac{1}{2}mR^2$

$$v = \sqrt{\frac{2mgh}{m + \frac{1}{2}m}} = \sqrt{\frac{4}{3}gh}$$

$$\vec{\tau} = (r_x \hat{i} + r_y \hat{j} + r_z \hat{k}) \times (F_x \hat{i} + F_y \hat{j} + F_z \hat{k})$$

$r_x F_y (\underbrace{\hat{i} \times \hat{j}}_{\hat{k}})$ $\quad$ $r_x F_z (\underbrace{\hat{i} \times \hat{k}}_{-\hat{j}})$

$\hat{j} \times \hat{i} = -\hat{k}$ $\quad$ $\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{k} = \hat{i}$ $\quad$ $\hat{k} \times \hat{j} = -\hat{i}$

use Right hand Rule.

Easier way to do this.

Use determinant. See Notes

$$\vec{r} \times \vec{F} = (r_y F_z - r_z F_y)\hat{i} - \hat{j}(r_x F_z - r_z F_x) + \hat{k}(r_x F_y - r_y F_x)$$

We'll do example next time.