Reminders 04-08-10:

-POW 9 Due Tuesday
-Quiz in Recitation next week

Objectives

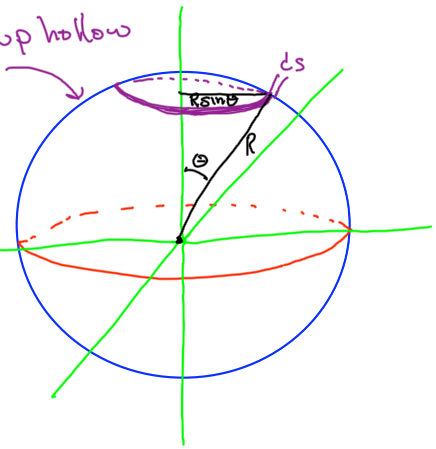
-Moment of Inertia Examples
-Torque + examples
-More on Rolling Motion

$I_{\text{rod center}} = \frac{1}{12} ML^2 \qquad I_{\text{ring}} = MR^2$

Find I for hollow sphere rotating about its c.m.

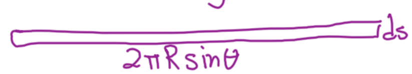
$$I = \int r^2\, dm$$

break up hollow sphere into rings



$$dI = dm\, r^2$$

CUT OUT RING



$ds = R\,d\theta$

$$dA = 2\pi R\sin\theta\, R\,d\theta = 2\pi R^2 \sin\theta\, d\theta$$

$\rightarrow r = R\sin\theta$

$$dm = \sigma\, dA = \frac{m}{4\pi R^2} 2\pi R^2 \sin\theta\, d\theta$$

$$I = \int R^2 \sin^2\theta \cdot \frac{m}{_2\cancel{4}\cancel{\pi}\cancel{R^2}} \cancel{2}\cancel{\pi}\cancel{R^2} \sin\theta\, d\theta$$

$$= \int_0^{\pi} \frac{R^2}{2} m \sin^3\theta\, d\theta$$

$$\int_0^{\pi} \sin^3\theta\, d\theta = \frac{4}{3}$$

$$I = \frac{mR^2}{2} \cdot \frac{4}{3} = \frac{2}{3} mR^2$$

Let's try solid sphere

$$I_{\text{hollow sphere}} = \frac{2}{3} m R^2$$

Break up solid into hollow spheres of thickness dr.

dI of each hollow sphere is $\frac{2}{3} dm\, r^2$

$$dI = \frac{2}{3} r^2 dm$$ need dm for each sphere

$$dm = \rho\, 4\pi r^2 dr = \frac{m}{\frac{4}{3}\pi R^3} 4\pi r^2 dr$$

$$dm = \frac{3 m r^2}{R^3} dr$$

$$I = \frac{2}{3} \int_0^R r^2 \frac{3 m r^2}{R^3} dr = \frac{2}{3}\left(\frac{1}{5}\right) 3 m R^2$$

$$= \frac{2}{5} m R^2$$

What is the rotational inertia of a hollow sphere rotated about an axis that passes through the end of a diameter?

$$I_p = I_{cm} + mR^2$$

$$= \frac{2}{3}mR^2 + mR^2 = \frac{5}{3}mR^2$$

What is the rotational inertia of a square plate rotated about an axis that passes through its corner?

$$h = \frac{\sqrt{2}L}{2}$$

$$I_p = I_{cm} + M\left(\frac{\sqrt{2}}{2}L\right)^2$$

$$I_{cm} = \frac{1}{6}ML^2$$

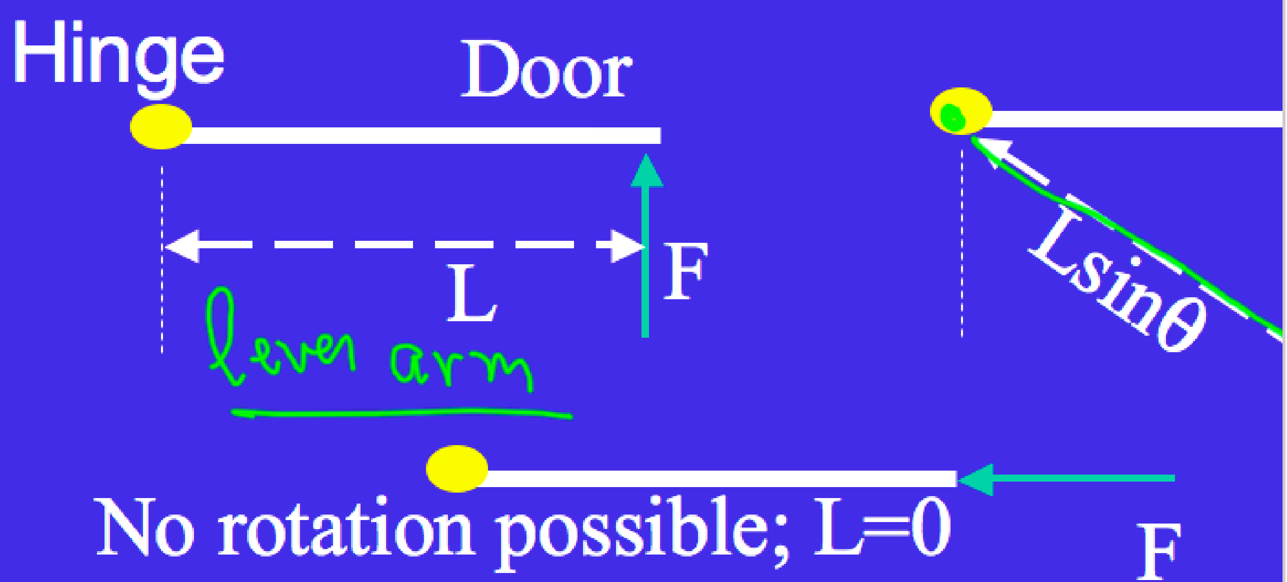
$$= \frac{1}{6}ML^2 + M\left(\frac{\sqrt{2}}{2}L\right)^2 = \frac{1}{6}ML^2 + \frac{1}{2}ML^2$$

$$\underline{= \frac{2}{3}ML^2}$$

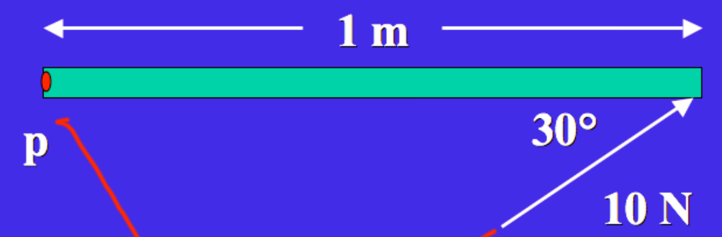
$$\boxed{F\ell = \tau}$$

Torque

- The ability of a force to rotate an ob depends on the distance from the pi (fulcrum) to force acting on the obje



What is the torque on the stick about p for the situation shown below?



$$\tau = I\alpha \qquad \ell = 1\,m \sin 30$$

$$\tau = (10\,N)(1\,m)\sin 30 = +5\,Nm$$

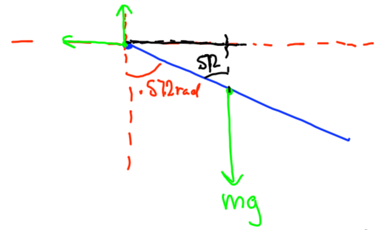
CCW

Now if $m = 12\,kg$ what is α?

$$\Sigma \tau = 5\,Nm = I\alpha = \tfrac{1}{3}ML^2\alpha$$

$$\alpha = \frac{5\,Nm}{\frac{1}{3}ML^2} = \frac{5}{\frac{1}{3}(12)(1)} = \frac{5}{4}/s^2$$

A 4 meter stick of mass m is suspended from an end and released from rest in a horizontal orientation. What is the angular acceleration of the stick when at an angle of 0.572 rad to the vertical (see examples 10.7-10.11)?



$$\Sigma = mg\frac{L}{2}\sin\theta = I\alpha = \frac{1}{3}mL^2$$

$$\cancel{m}g\frac{\cancel{L}}{2}\sin\theta = \frac{1}{3}\cancel{m}L^{\cancel{2}}\alpha$$

$$\alpha = \frac{3g}{2L}\sin\theta = 1.99/s^2$$

$$a_{cm} = \frac{L}{2}\alpha$$

Do this for case when θ 90°

$$\alpha = \frac{3}{2}\frac{g}{L}\sin\theta$$

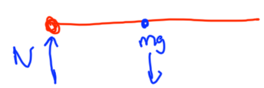
$$\alpha = \frac{3}{2}\frac{g}{L} \qquad a_{cm} = \frac{L}{2}\alpha = \frac{L}{2}\left(\frac{3}{2}\frac{g}{L}\right)$$

$$a_{cm} = \frac{3}{4}g$$

What is a_{end}?

$$a_{end} = L\alpha = L\left(\frac{3}{2}\frac{g}{L}\right) = \frac{3}{2}g$$

Now find force at pivot

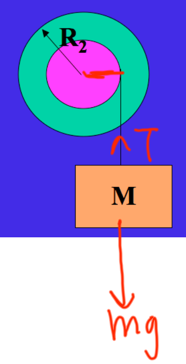


$$\Sigma F_y = N - mg = ma_{cm}$$

$$N - mg = -m\frac{3g}{4}$$

$$N = -m\frac{3g}{4} + mg = \frac{mg}{4}$$

A mass M is hung from a two-step pulley as shown below. The pulley is composed of two disks of radius R_1 and R_2 and rotational inertia I_1 and I_2, respectively. The string is wound around the pulley and the mass is released. Calculate the acceleration of the falling mass M (see example 10.12).



$$T - mg = -ma$$

$$TR_1 = (I_1 + I_2)\alpha$$

$$= (I_1 + I_2)\frac{a}{R_1}$$

$$T = (I_1 + I_2)\frac{a}{R_1^2}$$

$$(I_1 + I_2)\frac{a}{R_1^2} - mg = -ma$$

$$mg = a\left[m + \frac{I_1 + I_2}{R_1^2}\right]$$

$$a = \frac{mg}{m + \frac{I_1 + I_2}{R_1^2}}$$