

## **Reminders 04-06-10:**

- POW 9 Due Tuesday**
- Exam 3 Average 68%**

## **Objectives:**

- Rotational Kinematics**
- Rotational Energy**
- Moment of Inertia**
- Torque**

The angular position of a rotating object is given by  
 $\theta = 2t - 5t^2 + 2t^4$  rad. Find:

- the angular acceleration at  $t = 1$  s;
- the average angular acceleration between 1 and 2 s;
- the average angular speed between 1 and 2 s.

$$a) \frac{d^2\theta}{dt^2} = ?$$

$$\frac{d\theta}{dt} = \omega = 2 - 10t + 8t^3$$

$$\frac{d^2\theta}{dt^2} = -10 + 24t^2$$

sub.  $t=1$

$$\alpha = -10 + 24(1s) = +14/s^2$$

---

$$b) \alpha_{avg} = \frac{\Delta\omega}{\Delta t}$$

$$\omega(t=2) = 2 - 10(2) + 8(2)^3 = 2 - 20 + 64 = 46/s$$

$$\omega(t=1) = 2 - 10(1) + 8(1) = 2 - 10 + 8 = 0$$

$$\alpha_{avg} = \frac{46/s - 0}{1s} = 46/s^2$$

---

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

$$\theta(t=2) = 2(2) - 5(2)^2 + 2(2)^4 = 4 - 20 + 32 = 16$$

$$\theta(t=1) = 2(1) - 5(1)^2 + 2(1)^4 = 2 - 5 + 2 = -1$$

$$\omega_{avg} = \frac{16 - (-1)}{1s} = 17/s$$

A car with tires of radius of 25 cm comes to a stop from a 100 km/h (27.8 m/s) in 50 m without any **slipping** of the tires. Find:

- the angular acceleration of the wheels;
- the number of revolutions made in stopping.

Since object is rolling we  
Know  $a_T = a_{cm} = R\alpha$

$$V_i = 27.8 \text{ m/s} \quad V_f = 0 \quad \Delta x = 50 \text{ m}$$

Use  $V_f^2 = V_i^2 + 2a\Delta x$

$$0 = V_i^2 + 2a\Delta x$$

$$a_{cm} = \frac{-V_i^2}{2\Delta x} = \frac{-(27.8 \frac{\text{m}}{\text{s}})^2}{2(50 \text{ m})} = -7.78 \frac{\text{m}}{\text{s}^2}$$

$$\alpha = \frac{-7.78 \frac{\text{m}}{\text{s}^2}}{.25 \text{ m}} = -30.9 / \text{s}^2$$

b) Need  $\Delta\theta$

$$\Delta x = 50 \text{ m} \quad \Delta s = \Delta x = R\Delta\theta$$

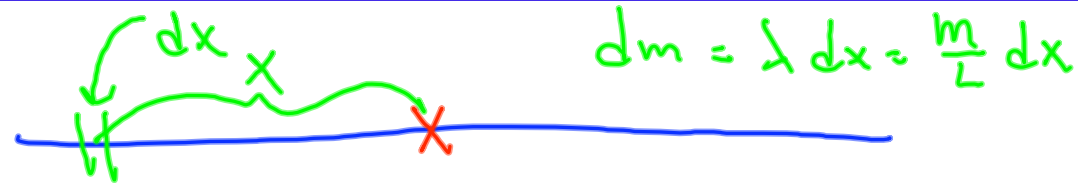
$$\Delta\theta = \frac{\Delta x}{R}$$

$$\begin{aligned} \# \text{ rev} &= \frac{\Delta\theta}{2\pi} = \frac{\Delta x}{2\pi R} = \frac{50 \text{ m}}{2\pi(.25)} \\ &= \underline{\underline{31.8 \text{ rev.}}} \end{aligned}$$

### Example:

What is the moment of inertia for a thin rod of length  $L$  and mass  $M$ , rotating on an axis through the center of mass of the rod and perpendicular to the rod.

PowerPoint (14)



$$\frac{M}{L} = \text{constant} = \lambda$$

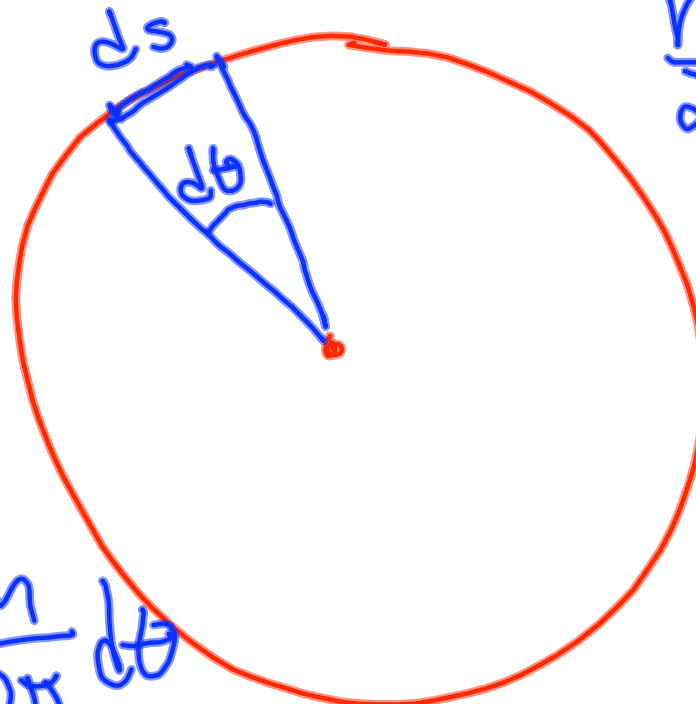
$$I = \int r^2 dm = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \frac{M}{L} dx$$

$$I = \frac{M}{L} \frac{x^3}{3} \Big|_{-\frac{L}{2}}^{\frac{L}{2}} = 2 \frac{M}{L} \frac{\left(\frac{L}{2}\right)^3}{3}$$

$$I = 2 \frac{M}{L} \left(\frac{L^3}{8}\right) \frac{1}{3} = \frac{1}{12} M L^2$$

**Examples: What is the moment of inertia for a ring of mass  $M$  and radius  $R$  rotating about an axis through its center?**

$$dm = \lambda ds$$



$$\frac{M}{2\pi R} = \lambda$$

$$ds = R d\theta$$

$$dm = \lambda R d\theta$$

$$dm = \frac{M}{2\pi R} R d\theta$$

$$= \frac{M}{2\pi} d\theta$$

$$I = \int_0^{2\pi} R^2 \frac{M}{2\pi} d\theta$$
$$= MR^2$$