

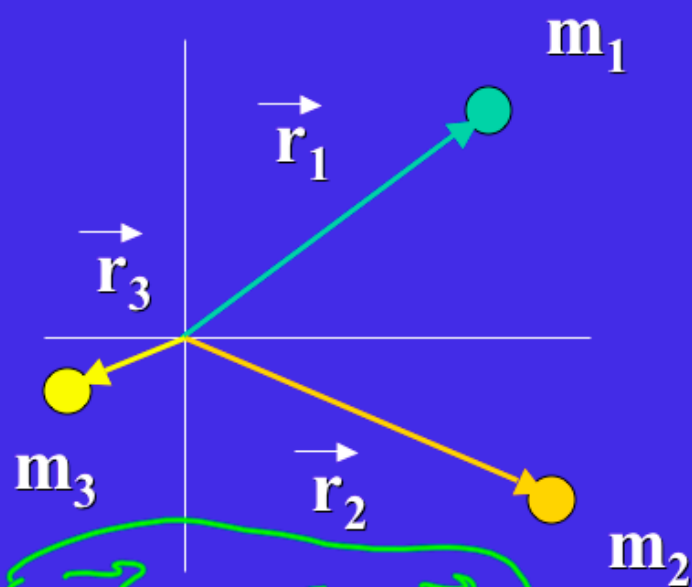
## **Reminders 03-16-10:**

- Quiz 7 on Power Work-Kinetic Energy Thm and Conservation of Energy in Recitation Next Week.**
- POW 7 Due Today**
- Short Quiz Thursday on Energy Level Diagrams.**
- Exam 3 Ch 7,8, and 9 March 25. No Makeups.**

## **Objectives:**

- Systems of Particles**
- Center of Mass**
- Conservation of Momentum**

→  
Now let's write  $\vec{a}$  as a second derivat



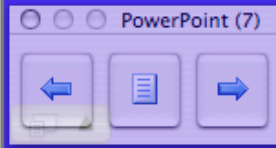
$$\vec{F}_{EXT} = \sum \frac{d\vec{p}_i}{dt} = \sum m_i \vec{a}_i$$

$$\vec{F}_{EXT} = M \frac{d^2 \vec{R}}{dt^2} = \sum m_i \vec{a}_i$$

only if

$$M \vec{R} = \sum m_i \vec{r}_i$$

$$\vec{R} = \frac{\sum m_i \vec{r}_i}{M}$$



→  
Now let's write  $\vec{a}$  as a second derivat

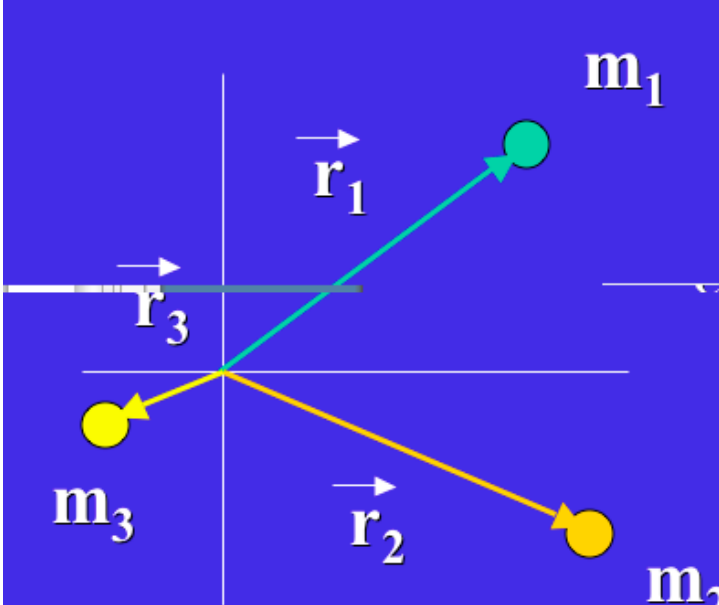


Diagram illustrating a system of three particles ( $m_1$ ,  $m_2$ ,  $m_3$ ) with position vectors  $\vec{r}_1$ ,  $\vec{r}_2$ , and  $\vec{r}_3$  relative to a common origin. The origin is labeled "center of mass".

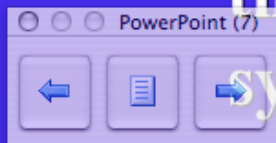
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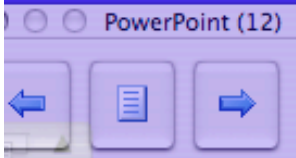
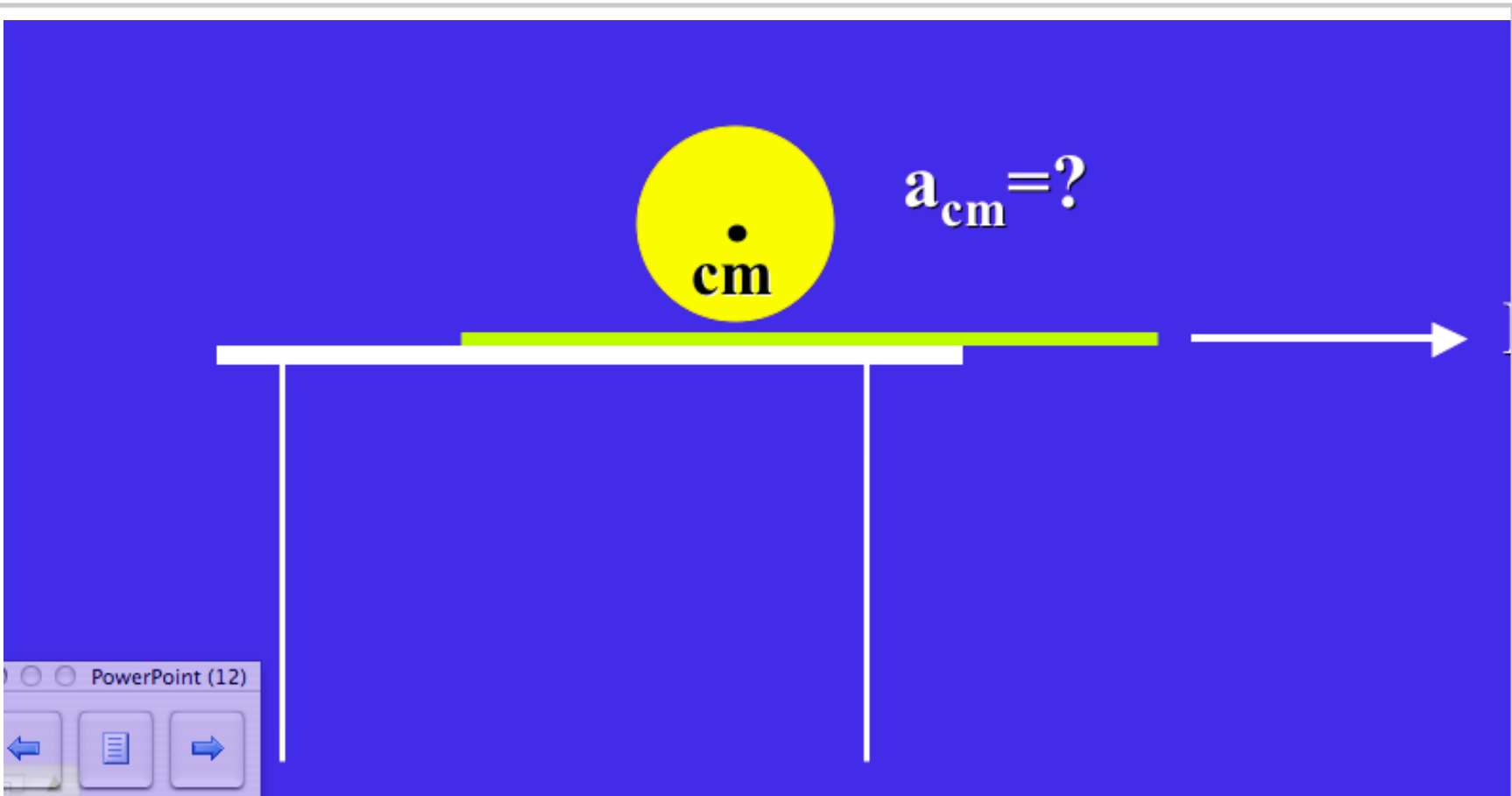
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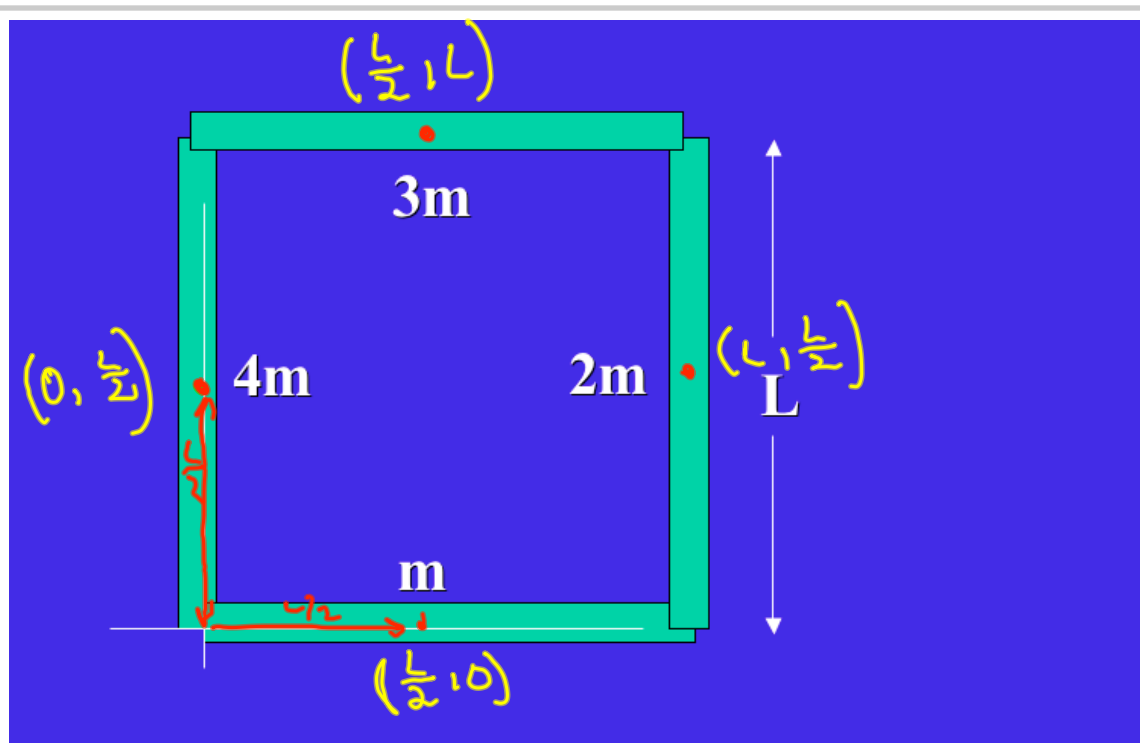
$$M \vec{R} = \sum m_i \vec{r}_i$$

→  
 $\vec{R}$  is called the center of mass of system  
think of it as the average position of a n  
system of particles.





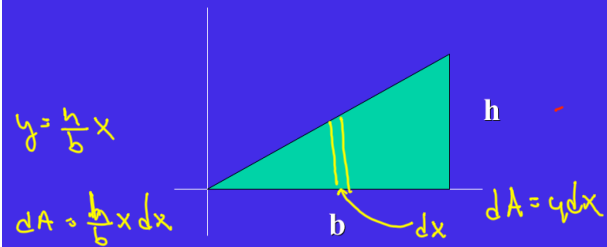
$a_{cm}$  to Right



$$X_{cm} = \frac{m(\frac{L}{2}) + 2m(L) + 3m(\frac{L}{2}) + 0}{10m} = 0.4L$$

$$Y_{cm} = \frac{0 + 2m(\frac{L}{2}) + 3m(L) + 4m(\frac{L}{2})}{10m} = 0.6L$$

$$\vec{r}_{cm} = (0.4\hat{i} + 0.6\hat{j})L$$

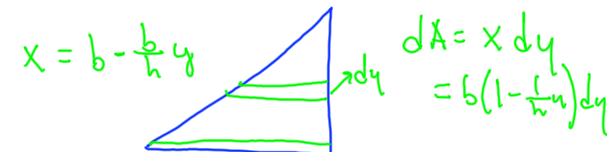


$$\vec{r}_{cm} = \frac{\int \vec{r} dm}{\int dm}$$

$$X_{cm} = \frac{\int x dm}{\int dm} = \frac{\int x \sigma dA}{\int \sigma dA}$$

$$X_{cm} = \frac{\int x dA}{\int dA} = \frac{\int_0^b x \frac{h}{b} x dx}{\int_0^b \frac{h}{b} x dx} = \frac{\int_0^b x^2 dx}{\int_0^b x dx}$$

$$X_{cm} = \frac{\frac{b^3}{3}}{\frac{b^2}{2}} = \frac{2}{3}b$$



$$Y_{cm} = \frac{\int y \sigma b(1 - \frac{y}{h}) dy}{\int_0^h \sigma b(1 - \frac{y}{h}) dy} = \frac{\int_0^h y(1 - \frac{y}{h}) dy}{\int_0^h (1 - \frac{y}{h}) dy}$$

$$Y_{cm} = \frac{\frac{y^2}{2} - \frac{y^3}{3h}}{y - \frac{y^2}{2h}} \Big|_0^h = \frac{\frac{h^2}{2} - \frac{h^3}{3h}}{h - \frac{h^2}{2h}} = \frac{\frac{h^2}{2} - \frac{h^2}{3}}{\frac{h}{2}} = \frac{\frac{h^2}{6}}{\frac{h}{2}} = \frac{h}{3}$$

$$\vec{r}_{cm} = \frac{2}{3}b \hat{i} + \frac{h}{3} \hat{j}$$