Reminders 03-04-10:

-Quiz 6 on Chapter in Recitation Next week.

-Video lectures http://ocw.mit.edu/OcwWeb/Physics/8-01Physics-IFall1999/CoursesHome/index.htm

-Since Centripetal Force Lab is our fourth experiment, it is the last lab with free pre-check.

Objectives:

-Work-Kinetic Energy Theorem

-Power

-Conservative Forces

-Potential Energy

Work Review

What is the most general expression for work? It is the one that works in all cases!!

$$W = \int \vec{F} \cdot d\vec{r}$$

Mathematically speaking, the dot product is the projection of one vector onto another vector. In our case, it is the projection of F onto the axis defined by the displacement vector.

To find the work done by the net force we can add up the work done by each force on an object; or we can calculate the work done by the net force and "dot" it with the displacement vector.

Positive work done by a force on an object means that the force wants to accelerate the object, while negative work done by a force on an object means that the force wants to slow down the object.

What is the work done by a force $F=3x^2$ directed in the x-direction and applied from x=1m to x=5m?

$W = \int \vec{F} \cdot d\vec{r}$ $\quad \vec{F} = 3x^2 \hat{i}$

$d\vec{r} = \hat{i}\, dx$

$\vec{F} \cdot d\vec{r} = 3x^2 \hat{i} \cdot \hat{i}\, dx$

$= 3x^2 dx$ $\quad W = \int_1^5 3x^2 dx = x^3 \Big|_1^5$

$W = 125J - 1J = 124J$

A block that is on a table (not frictionless) is pushed to the left by a force equal to 5N. The block moves to the left at a constant speed of 2m/s. We can conclude that the total work done by all forces acting on the object is

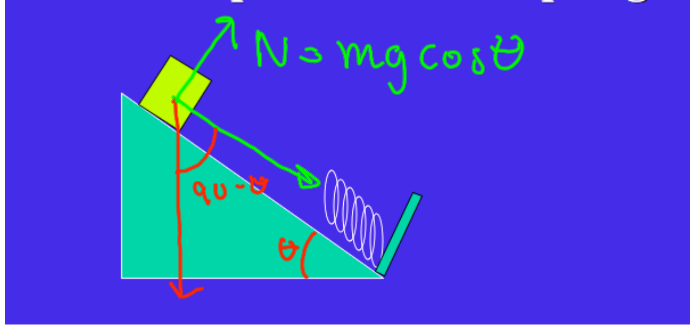
a. greater than zero.

b. less than zero.

c. equal to zero.

d. unknown.

compression of the spring?



$W = \Delta K$ since $K_i = K_f = 0$

$W_{net} = 0$

$W_g + W_f + W_s = 0$

$$mg(l+x)\cos(90-\theta) + \mu mg\cos\theta(l+x)\cos 180 - \tfrac{1}{2}kx^2 = 0$$

$$(l+x)\cos(90-\theta) - \mu\cos\theta(l+x) - \frac{1}{2}\frac{k}{mg}x^2 = 0$$

$$(1+x)\cos 60^\circ - 0.2\cos 30(1+x) - \frac{50}{9.8}x^2 = 0$$

simplify then

use quadratic formula

$x = 2.2\,m$

$$\text{Power} = \frac{dW}{dt}$$

$$W = \int \vec{F} \cdot d\vec{r}$$

$$P = \frac{d}{dt} \int \vec{F} \cdot d\vec{r} = \frac{d}{dt} \int (\vec{F} \cdot \vec{v})\, dt$$

$$P_{inst} = \vec{F} \cdot \vec{v} \; ; \; P_{avg} = \vec{F} \cdot \vec{v}_{avg}$$

If W represents work done by net force, then this is equal to ΔK

$$P = \frac{dK}{dt} \quad \text{(delivered by net force)}$$

A 10 N horizontal force is applied to a 10 kg block resting on a frictionless, horizontal surface. The block starts from rest.
What is the average power supplied by the force from t = 0 to t = 3 s?
What is the instantaneous power due to the force at t = 3 s?

$$P_{avg} = \frac{W}{\Delta t} = (10N)\frac{\Delta x}{\Delta t} = (10N)V_{avg}$$

$$a = \frac{F}{m} = 1\,m/s^2 \qquad V_0 = 0$$

$$V_f = at = 3\,m/s$$

$$V_{avg} = \frac{3+0}{2} = 1.5\,m/s$$

$$P_{avg} = (10N)(1.5\,\tfrac{m}{s}) = 15W$$

$$P_{inst} = (10N)(3\,m/s) = \underline{30W}$$

$$P = F_x v_x = m a_x v_x = \frac{dK}{dt} \Rightarrow P\Delta t = \Delta K$$

$$a_x = \frac{P}{m v_x} \qquad x(t) = ??$$

$$\frac{dv}{dt} = \frac{P}{m}\frac{1}{v} \; ; \; v\,dv = \frac{P}{m}dt$$

$$\frac{1}{2}v^2 = \frac{P}{m}t$$ let constant of int. be zero

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = \frac{P}{m}t \qquad \left(\frac{dx}{dt}\right)^2 = \frac{2P}{m}t$$

$$\frac{dx}{dt} = \sqrt{\frac{2P}{m}t} \qquad \int dx = \int \sqrt{\frac{2P}{m}t}\,dt$$

$$X = \sqrt{\frac{2P}{m}}\,t^{3/2}\left(\frac{2}{3}\right) = \sqrt{\frac{8P}{9m}t^3}$$

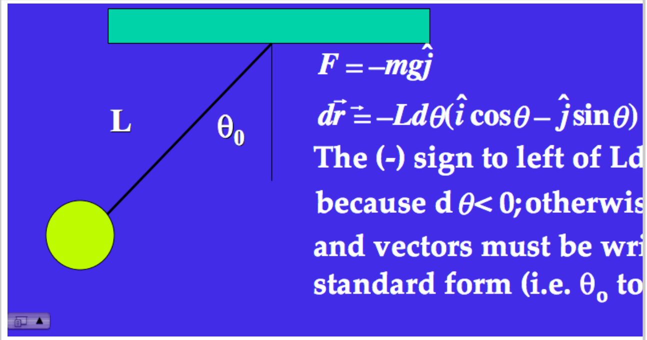
Review

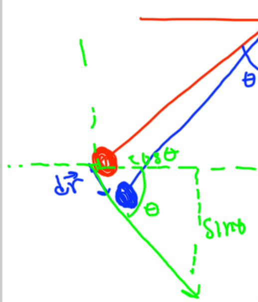
Do constraint forces do work?

T/F kinetic energy depends on referen

If m=2kg and v=3i+4j, KE=..?

$$\frac{1}{2}(2kg)(\vec{v}\cdot\vec{v}) = \underline{25J}$$





$|d\vec{r}| = Ld\theta$

$\hat{r} = \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

notice
$d\theta < 0$

$$d\vec{r} = L(-d\theta)[\cos\theta\,\hat{i} - \sin\theta\,\hat{j}]$$

$$\vec{F}\cdot d\vec{r} = -mg\hat{j}\cdot L(-d\theta)[\cos\theta\,\hat{i} - \sin\theta\,\hat{j}]$$

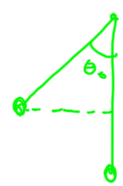
$$= -mg\sin\theta\, L\,d\theta$$

$$W = \int_{\theta_0}^{0} -mg\sin\theta\, L\,d\theta$$

$$= mgL\int_{\theta_0}^{0} -\sin\theta\, d\theta$$

$$= mgL\cos\theta\Big|_{\theta_0}^{0}$$

$$= mgL(1-\cos\theta_0)$$



$L(1-\cos\theta_0)$

$L - L\cos\theta_0$

change in y

$$W_c = \int_a^b \vec{F}_c \cdot d\vec{r} = f(b) - f(a)$$

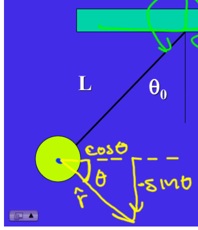
let

$$\int_a^b \vec{F}_c \cdot d\vec{r} = U_a - U_b = -\Delta U$$

call U potential energy

Example:

A pendulum of mass m and length L is released from rest at an angle θ_0. How much work is done by gravity in falling to the equilibrium position? Find $v(\theta)$ for a simple pendulum (note $d\theta < 0$!) Can we use Newton's 2nd law for the pendulum?



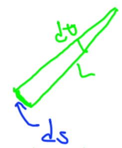
$\vec{F} = -mg\hat{j}$

$d\vec{r} = -Ld\theta(\hat{i}\cos\theta - \hat{j}\sin\theta)$

The (-) sign to left of $Ld\theta$ is there because $d\theta < 0$; otherwise, angles and vectors must be written in standard form (i.e. θ_o to 360°).

$W = \int \vec{F} \cdot d\vec{r}$

$ds = Ld\theta$



$\vec{F} = -mg\hat{j}$

$d\vec{r} = ?$

$d\vec{r} = -Ld\theta$ (unit vector in direction of motion)

$= -Ld\theta(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$

$\theta = 270 - \theta_{\text{standard}}$

$d\theta = -d\theta_{\text{standard}}$

$dW = -mg\hat{j} \cdot \left[-Ld\theta(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})\right]$

$= -mgL\sin\theta\, d\theta$

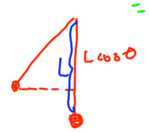
$\int dW = \int_{\theta_o}^{\theta_f} -mgL\sin\theta\, d\theta$

$= mgL\cos\theta \Big|_{\theta_o}^{\theta_f}$

$= mgL(\cos\theta_f - \cos\theta_o)$

if $\theta_f = 0$ then

$mg\underline{L(1-\cos\theta_o)}$



$\Delta y = L - L\cos\theta$